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A THREE-DIMENSIONAL SPECTRAL CLASSIFICATION OF G AND K STARS

BY

KJELD GYLDENKERNE

København 1964
Kommissionær: Ejnar Munksgaard

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Contents

Synopsis

On the basis of three classification indices k, n and m, related to the break around the H- and K-lines, to the cyanogen absorption at 4200 A and to a broad absorption of the continuum, a system of three independent classification parameters S, L and C has been established for stars of spectral types G 5 – K 3.5 and luminosity classes II–III to IV. The parameter S is closely related to average MK spectral types, parameter L to the luminosity, and C is a third parameter which measures the remaining dispersion and is probably related chiefly to the chemical composition. With systematic dependence also on S, absolute magnitudes $M_v(k, n, m)$ have been derived from L on the basis of M_v given by trigonometric parallaxes (main-sequence stars), cluster parallaxes (Hyades) and M_v(K) determined by the Wilson-Bappu method for a large body of giant stars. The $M_v(k, n, m)$ and M_v(K) values agree with a scatter which corresponds to a mean error of $\pm 0\overset{m}{.}46$ for each of the two sets of absolute magnitudes.

The classification parameters have been compared with the Cambridge intensity ratios and with photoelectric colour indices; furthermore the effect of duplicity has been investigated. The distribution of the parameters and the galactic space-velocity components on the program stars have been studied. It was found that the data of about 160 giants and subgiants in the considered range can be divided into two groups according to C with a limit not much different from the average C value. One of these groups contains young giants evolved from A-type main-sequence stars as well as older stars evolved from the F- and G-type section. The other group includes practically no young giants. This division is also reflected in the velocity distribution of the two groups and is similar to the earlier division of Vyssotsky who used strong-line and weak-line criteria.

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1. Introduction

Quantitative classification of stellar spectra has been carried out extensively during the recent years. A survey of the different methods is given by STRÖMGREN (1963b). For the early type stars, reference should be made to the work of CHALONGE and collaborators (cf. CHALONGE 1958), STRÖMGREN (1956a, 1956b, 1958a, 1958b), PETRIE (1956), CRAWFORD (1958) and T. and J. H. WALRAVEN (1960). Relations between precise classification equivalents of main-sequence B, A and F stars, stellar ages and space velocities have been discussed recently by STRÖMGREN (1962b, 1963a).

The present treatise is concerned with the classification of G and K stars. For these stars we mention the work by HOSSACK (1954), by HALLIDAY (1955) and by OKE (1957, 1959), who have developed methods for measuring line-depth ratios in photographic spectrograms of 33 A/mm dispersion at H_γ. WILSON and BAPPU (1957) used spectra with a dispersion of 10 A/mm and found that the width of the emission component of the H- and K-lines of ionized Ca was a sensitive luminosity indicator for the G, K and M stars, independent of the spectral type; the absolute magnitudes $M_v(K)$ have been calibrated for a number of bright stars (WILSON, 1959).

Improved observational methods have shown the influence of a third parameter, e. g. the cyanogen absorption at 4200 A is affected as could be found through the visual inspection method in the Atlas of MORGAN, KEENAN and KELLMAN (1943). It has been further confirmed by ROMAN (1952) in a classification of a large number of bright G and K stars; she also noticed other differences denoted as strong-line and weak-line characteristics in these spectra. These features are most probably due to differences in the abundances of the metals and of the O, C, N group relative to hydrogen, according to SCHWARZSCHILD, SPITZER and WILDT (1951), and are found to be related to differences in space-velocity characteristics (ROMAN 1952, KEENAN and KELLER 1953, KEENAN 1958, VYSSOTSKY and SKUMANICH 1953, BLAAUW 1958).

The cyanogen absorption is apparently a sensitive population effect and has been measured quantitatively in photographic spectra by KEENAN (1958) and by YOSS (1961, 1962) and photoelectrically using narrow-band techniques by STRÖMGREN and GYLDENKERNE (1955) and GYLDENKERNE (1958a, 1958b, referred to as Papers I and II in what follows), by GRIFFIN and REDMAN (1960) and by CRAWFORD (1961). However, already from objective prism observations initiated more than 40 years ago by LINDBLAD (1922) and carried out extensively at the Stockholm and Uppsala observatories, the cyanogen absorption is known to depend on the luminosity and to a lesser degree on the spectral type. Therefore, in addition to a cyanogen equivalent, criteria for the spectral type and luminosity should also be determined accurately.

Photoelectric narrow-band measurements of different outstanding features in the G and K type spectra have been made by astronomers at the Cambridge observatory in order to test the sensitivity of these features to the stellar parameters. In addition to the cyanogen observations, GRIFFIN and REDMAN (1960) have published measurements of the G-band, the magnesium b triplet at 5170 A has been measured by DEEMING (1960), and GRIFFIN (1961) has made measurements of the FeI-lines at 5250 A and of the sodium D-lines.

In the method of STRÖMGREN and GYLDENKERNE (1955), three classification indices, similar to three of the Stockholm-Uppsala equivalents were measured by means of interference filters. These quantities are k, which is related to a region close to the K-line of ionized Ca, n, which measures the cyanogen absorption at 4200 A and g, which measures the break at the G-band. The three indices provided a two-dimensional classification, but the analysis of the material of about 250 bright G and K stars clearly showed the effect of a third parameter. BORGMAN (1959) measured the metallic-line index m as defined by STRÖMGREN (1958a) for a few G and K stars and found a separation of high-velocity giants from low-velocity giants in an m, spectral type diagram. Borgman then suggested that it might be possible to separate the three parameters using m together with two of the indices determined by STRÖMGREN and GYLDENKERNE. In 1960–61 the present author measured m for almost all stars for which k, n and g were already available. In a preliminary analysis (GYLDENKERNE 1961) it was found that the spectral type, the luminosity and a third parameter can be determined for G8 to K3 giants by means of k, n and m.

Since the late-type spectra are rich in detail, it should be expected that differences in the spectral transmission regions of the classification indices would influence the sensitivity of the indices to the principal stellar parameters. For the cyanogen index the dependence on the wave length regions has been convincingly demonstrated by YOSS (1962) with reference to the CN ratio measured by GRIFFIN and REDMAN, the present index n and a similar index measured by CRAWFORD (1961). The latter index utilizing filters narrower than n with somewhat different peak wave lengths appeared to be considerably more sensitive for the same measuring accuracy. Similar results may be found for the other indices. Furthermore, the physical significance of an index is not quite clear in every case. For the index k, as an example, it may be that effects other than the K-line contribute considerably to the variation of the index with the stellar parameters (cf. p. 28). In this connection we refer to results obtained by VAN DEN BERGH (1963) for the quantity Δ, covering almost the same spectral range as k.

Further investigations of the dependence of the indices on the spectral transmission regions are in progress at this observatory. However, since through the preliminary analysis it developed that k, n and m, as originally defined, reproduce spectral types and absolute magnitudes in a simple way with good accuracy and furthermore a third parameter of significant variation appears, we have carried this analysis as far as possible for the available data of G and K stars. The results concerning properties of the classification system as well as its relation to age and kinematics of the program stars are presented in this paper.

The measurements of the index m and its relation to spectral types and luminosity classes are described in Section 2. The three-dimensional classification system is established in Section 3. In Sections 4 and 5 the classification parameters are related to other spectrophotometric quantities and to photoelectric colour indices. The properties of the third parameter are discussed in Section 6, and the relations of the three parameters to the space velocity and its galactic components are studied in Section 7. In the final section the results of the analysis are summarized, and the projected extension of the classification program for late type stars is outlined.

2. The Metallic-Line Index *m* for G and K Stars and its relation to MK Spectral Types and Luminosity Classes

The observations of the index m have been carried out by means of a two-beam photometer attached to the 20 inch reflecting telescope of the Brorfelde observatory. The photometer, constructed in the observatory workshop under the direction of P. Bechmann, follows a principle given by Strömgren and quoted by Crawford (1958, Fig. 3, second version). The two beams from the beam splitter pass two field lenses, which image the telescope objective on the thin layers of two interference filters. These images are projected upon the photocathodes of two photomultipliers by a second set of lenses. The anode currents are measured utilizing integrators which were developed and constructed by R. H. Weitbrecht (1957). A programming unit permits an automatic recording of the two voltages—one after the other—on a one-pen recorder, Philips PR 2210 A/21. Two Lallemand 20 stage photomultipliers Gabriel VIII and Norbert VIII have been used without refrigeration throughout the program.

The three intensities were measured through interference filters made by Schott and Genossen, Mainz. These PIL-type filters had the following characteristics:

Wave length of maximum transmission	4050 A	4510 A	4970 A
Width corresponding to one half of maximum transmission	75 A	80 A	90 A
Width corresponding to one tenth of maximum transmission	230 A	250 A	260 A

The maximum transmission of the filters was about 40 per cent.

These filters defined the colour index difference m as follows:

$$m = \text{const.} - 2.5 \left\{ [\log I(4970) - \log I(4510)] - [\log I(4510) - \log I(4050)] \right\}$$

The 4050 filter and one 4510 filter were placed in filter holder I (transmitted beam). Another 4510 filter identical with the first one was placed in filter holder II (reflected beam), which also contained the 4970 filter.

The measurements were carried out according to the following scheme:

I	II	
4050	4510	star + sky
4510	4970	star + sky
4050	4510	star + sky
	etc.	
4050	4510	sky
4510	4970	sky

This procedure eliminated effects of varying relative sensitivity of the two parts of the photometer. The number of deflections depends mainly on the quality of the sky. The extinction could vary from one measurement of a pair to the next one, but this effect is practically eliminated by alternating the two colour indices, which almost have equal wave length bases. The integration times varied from 10 to 40 sec for the stars in this program (mainly brighter than $5^{m}_{\cdot}5$).

The observations have been corrected to zenith utilizing an average extinction coefficient, 0.039. Night corrections have been determined from standard star measurements close to zenith each night. The results have been reduced to approximately the same zero point as the one used by Strömgren (1958a) for the F stars. Comparison of measurements of the same stars obtained on different nights yields a mean error of one single observation of $\pm 0^{m}_{\cdot}008$ (mean errors are used throughout in this paper).

The measurements obtained in the described manner have been tested by observing a certain number of the program stars by means of an ordinary single channel photometer together with the three filters. The two sets of observations are in satisfactory agreement.

The values of m for 233 G and K stars and a few stars of other spectral types are given in Table 11.

The relation of the index m to the MK spectral types and luminosity classes is shown in Fig. 1. For stars of class III, mean values of m have been derived and are given in Table 1. The mean values are represented by large dots in Fig. 1*. In the same table are also listed the mean values of k, n and g from Table 1 of Paper II.

Table 1.

Mean values of classification indices for stars of luminosity class III.

MK	$\overline{m}$	number of stars	$\overline{k}$	$\overline{n}$	$\overline{g}$
G8 III	0.354	37	0.342	0.171	0.258
K0 III	0.411	50	0.293	0.198	0.286
K1 III	0.465	14	0.246	0.218	0.312
K2 III	0.522	28	0.206	0.250	0.341
K3 III	0.614	26	0.150	0.284	0.391
K4 III	0.708	8	0.114	0.298	0.432
K5 III	0.790	8	0.105	0.299	0.476

* The dots correspond to star numbers slightly different from those of the table.

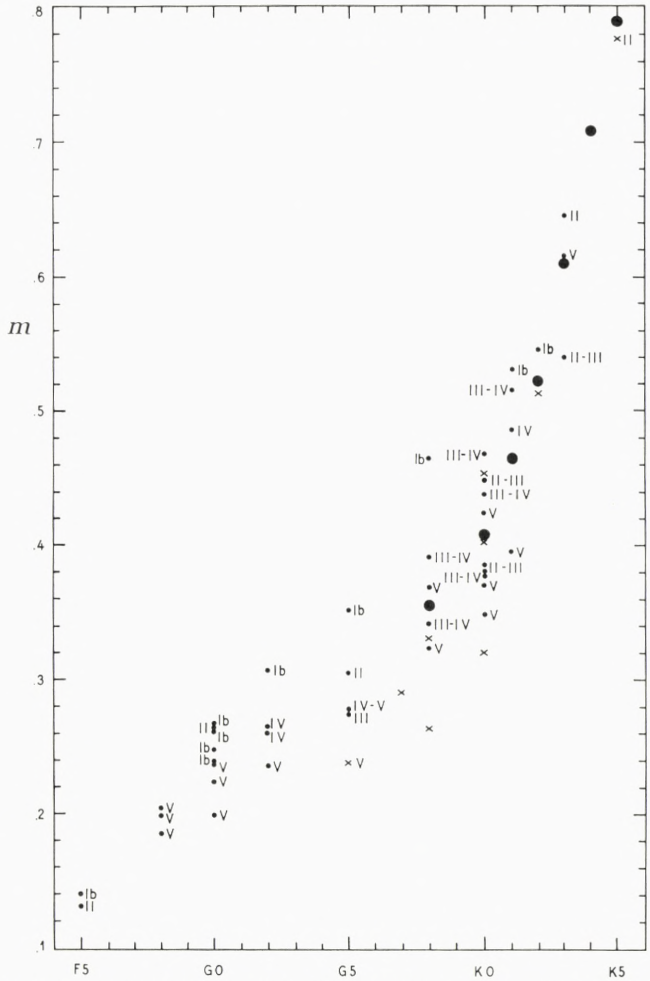

Fig. 1. The metallic-line index m as function of the MK spectral type and luminosity class. Crosses indicate high-velocity stars.

It is seen that the dependence on spectral type is more pronounced for m than for k and g in the whole range G8–K5; the sensitivity of m exceeds that of k and g by a factor of 1.2–2.6.

For the program stars we found the same internal accuracy, $\pm 0\overset{m}{.}008$, of m as of k, n and g. For the mean values of these quantities we provisionally adopted the external mean error $\pm 0\overset{m}{.}010$ in Paper II. Although we have determined m in only one series of measurements we shall consider the mean error $\pm 0\overset{m}{.}010$ as representative for all four quantities in what follows.

Using this mean error we find the accuracy of spectral types determined from m as shown in Table 2. As in earlier cases we have assumed that the range G8 III to K0 III covers one spectral subtype.

TABLE 2.
Mean errors of spectral subtypes determined from m
(unit one spectral subtype).

G8 III – K0 III	±0.18
K0 III – K1 III	±0.19
K1 III – K2 III	±0.18
K2 III – K3 III	±0.11
K3 III – K4 III	±0.11
K4 III – K5 III	±0.12

3. The Three-Dimensional Classification System

In principle it would be desirable to carry out a classification similar to the present one by relating the classification equivalents to computed or measurable stellar atmospheric parameters, such as a measure of the colour temperature, the absolute magnitude and the chemical composition. However, colour equivalents in the normal spectral region as the colour indices $(U-B)$ and $(B-V)$, are expected to be influenced by the abundance effect for the spectral types considered here, and long wave length colour indices are thus far available only for a small number of our stars. Absolute magnitudes based on trigonometric parallaxes are generally quite uncertain for stars in the giant region, and only a few group stars are included in the present program. Abundances, finally, have been published for extremely few G and K giants. Therefore, this analysis has been carried out by separating three independent parameters S, L and C, so that S is closely correlated with average MK spectral types and L with the absolute magnitudes M_v, determined by the method of WILSON and BAPPU (1957), or by the group parallax for the Hyades and the trigonometric parallax for the main-sequence stars; the remaining dispersion defines the third parameter C. An approximate, linear and semi-graphical method has been used.

Figs. 2 and 3 show the distribution of the stars in an (n, k) diagram and a (k, m) diagram. The luminosity class is indicated for the stars of classes Ib to II and IV to V. Dots without indication correspond to stars of luminosity classes II–III, III and III–IV; the large dots correspond to the mean values in Table 1. Crosses indicate stars of high velocity according to KEENAN and KELLER (1953). Both diagrams demonstrate clearly a two-dimensional distribution with the supergiants and main-sequence stars lying on each side of the class III stars. However, the dispersion of the latter stars around a line through the mean points is considerable. Some class III stars lie close to the supergiants, while others lie close to the main-sequence stars. This is very unlikely due to an error in the MK luminosity classification, but should probably be ascribed to the variation of a third parameter, which is assumed to be the initial chemical composition. Extreme abundance-ratio values have been found

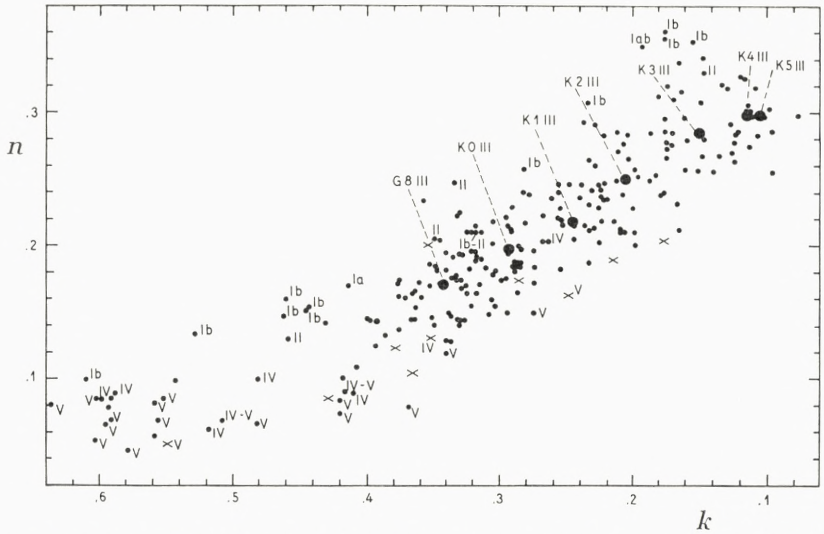

Fig. 2. The cyanogen-absorption index n plotted against the index k. Crosses indicate high-velocity stars.

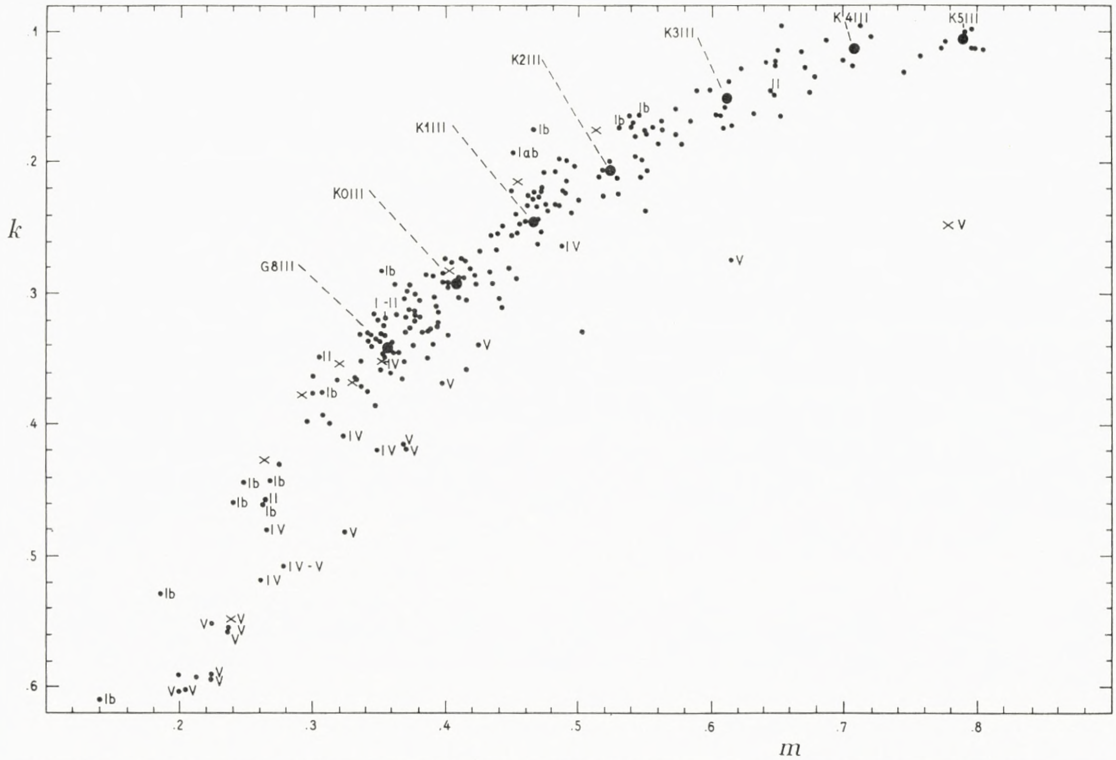

Fig. 3. The index k plotted against the metallic-line index m. Crosses indicate high-velocity stars.

for some class III high-velocity stars, and therefore, these stars should exhibit large deviations from the mean class III stars in the (n, k) and (k, m) diagrams. In the (n, k) diagram the extreme positions of the high-velocity stars (marked by crosses) close to the main-sequence stars have been noticed in Paper II. It is then significant that crosses corresponding to the same high-velocity stars lie close to the supergiants in the (k, m) diagram. This suggests the possibility of separating the chemical composition effect from the luminosity effect by means of the three classification indices.

We have derived the three classification parameters S, L and C by means of the (n, k) diagram and the (k, m) diagram, in the following manner.

In both diagrams we consider the variation along the line through the mean points as related to a first parameter, a measure of the spectral type. Since the mean points in the (n, k) diagram very nearly define a straight line, we determine the quantity giving the spectral type, as a function of n and k:

$$\text{Spectral-type parameter } S = n - 1.70\,k + 1.000\,.$$

With the adopted mean error $\pm 0^{\text{m}}.010$ of n and k and the correlation coefficient -0.83 between them, the mean error of an S value is $\pm 0^{\text{m}}.026$. The mean values of S for different spectral types of luminosity class III are given in Table 3. Accordingly, the mean error of S corresponds to a classification accuracy of one quarter of a spectral subtype from G8 to K3.

Table 3.
Mean values of the spectral-type parameter.

	$\bar{S}$
G8 III	$0^{\text{m}}.590$
K0 III	0.700
K1 III	0.800
K2 III	0.900
K3 III	1.029
K4 III	1.104
K5 III	1.121

The second parameter in the two diagrams is measured as the distance from the mean line plus an arbitrary constant. From the (n, k) diagram this parameter x can be expressed through the relation $x = 0.862\,n + 0.507\,k$. For the (k, m) diagram we have determined the distance graphically and adopted the second parameter y as the distance plus 0.100. In the preliminary investigation the material was divided into two groups, corresponding to the intervals in spectral type G8–K1 and K2–K3, and for each part a linear relation between k and m was used. In the present case we have approximated the slightly curved (k, m) relation piecewise by straight lines.

Approximate linear relations have been derived for y as $y = 0.75\,k + 0.66\,m - 0.39$ from G8 to K1, and $y = 0.86\,k + 0.52\,m - 0.35$ from K2 to K3. The accuracy of the different parameters related to the classification indices, and the scatter in relations between the parameters and other quantities are estimated using an average relation $y = 0.80\,k + 0.60\,m + \text{const}$ and the above relations for x and S. The effect of the correlation moments has been determined by means of correlation coefficients, computed as $\varrho(n, k) = -0.83$, $\varrho(m, k) = -0.64$ and $\varrho(n, m) = +0.54$. Thus, according to the adopted mean error $\pm 0^{\mathrm{m}}.010$ of k, n and m, the mean errors of x and y are found to be $\pm 0^{\mathrm{m}}.005$ and $\pm 0^{\mathrm{m}}.006$ respectively.

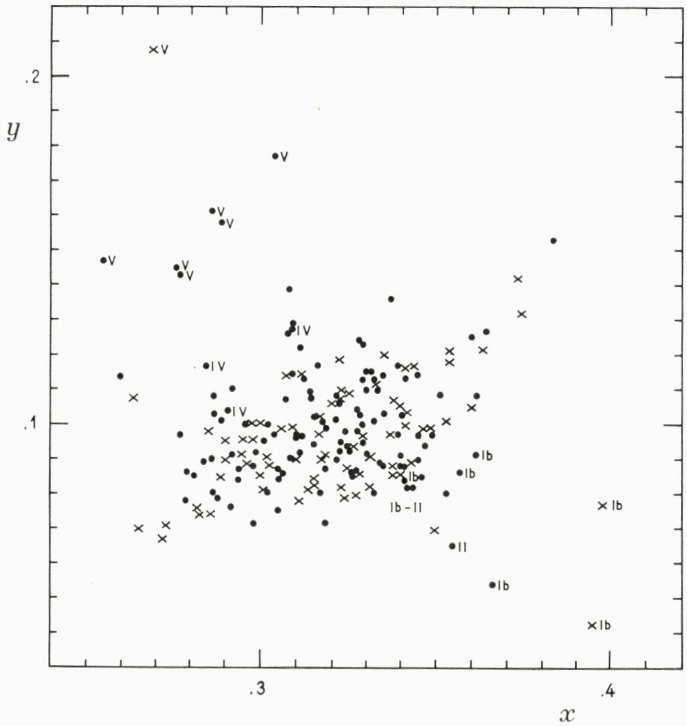

Fig. 4. The dispersion y around the luminosity class III mean-line in the (k, m) diagram, plotted against the corresponding dispersion x in the (n, k) diagram (x and y are both independent of the spectral type). The data are divided into two groups according to the spectral type parameter S. $S < 0.80$, dots; $S > 0.80$, crosses.

In Fig. 4, y is plotted against x. The material is divided into two groups: dots correspond to the region $0.35 < S < 0.80$ and crosses to $0.80 < S < 1.07$. These notations are used throughout in what follows unless otherwise stated. The diagram clearly exhibits a two-dimensional distribution of the stars, and both coordinates x and y are independent of the first parameter S.

Obviously the dispersion in the direction upper left to lower right corner of Fig. 4 is correlated with the luminosity. Accordingly, instead of x and y as second and third parameters we could use $x - y$ and $x + y$, and then $x - y$ would be a luminosity parameter (adopted in the preliminary discussion). However, improved parameters may be obtained as $x - qy$ and $x + 1/q\; y$ if absolute magnitudes are considered.

We have attempted an estimate of q considering stars having approximately the same absolute magnitude but differing significantly with respect to the third parameter. We concentrate the attention on stars with $M_v(K)$ in the range from $0^m_.0$ to $1^m_.0$, and divide this material into the two groups according to S, each group containing 17 stars. Separate plots of x versus y indicate linear relations for each group and by the least-squares method (errors both in x and y) we find $q = 1.60$ for $S < 0.8$ and $q = 1.89$ for $S > 0.8$. We have made another estimate of q by correcting all the considered giant and subgiant stars to the same absolute magnitude $M_v(\mathrm{K}) = 0.0$. The corrections are made by means of linear relations between M_v and x and y respectively, derived by means of the supergiants and the main-sequence stars. Again we divide the material into two groups and find for $S > 0.8$ the corrections $\Delta x = 0.011\,\Delta M_v$ and $\Delta y = -0.014\,\Delta M_v$, which are used for both groups (for $S < 0.8$ no supergiant can be used). Through least-squares computations of linear relations between the corrected x and y values we find $q = 1.07$ for $\mathrm{S} < 0.8$ (47 stars) and $q = 2.08$ for $\mathrm{S} > 0.8$ (22 stars). These results indicate a q larger than unity. We have used $q = 1.4$, and thus we define:

$$\text{Luminosity parameter } L = x - 1.40\,y$$
$$\text{Third parameter } C = x + 0.71\,y.$$

In Fig. 5, L is plotted against absolute magnitudes, and the data are divided into two groups with the above-mentioned symbols (the only K3 V star is included in the cross group in this case and also in the above computations of Δx and Δy). The supergiants of the dot group, mainly of the classes G0 Ib–G2 Ib, are omitted, and for the remaining data L appears to be linearly related to the absolute magnitudes. A least-squares solution yields the relation $M_v = 7.61 - 36.1\,L$ for $S < 0.8$, and $M_v = 6.20 - 32.6\,\mathrm{L}$ for $S > 0.8$. Thus the L coefficient as well as the constant in the relation varies with S. Adopting the mean value 34.3 for the coefficient, the constant will be $p = M_v + 34.3\,L$, which appears to be linearly correlated with S according to the plot in Fig. 6. Accordingly the absolute magnitude should be represented by a relation of the form $M_v = \mathrm{a} + \mathrm{b}S + \mathrm{c}L$. A least-squares solution for the stars in the range G5–K3.5 provides the relation

$$\begin{aligned} M_v = \; & 8.28 - 2.28\,\mathrm{S} - 32.3\,L \\ & \pm 0.31 \pm 0.42 \quad \pm 1.6. \end{aligned}$$

From the residuals between $M_v(k, n, m)$ computed from this formula and the given M_v values, we find the mean square of the scatter to be 0.43 on the basis of 69 stars of luminosity classes II–III to IV. The mean error of one single determination of $M_v(\mathrm{K})$ is estimated by Wilson to be $\pm 0^m_.4$. The average number of observations in the utilized $M_v(\mathrm{K})$ material is 1.8 (Wilson, privately communicated data), and thus the mean error of an $M_v(\mathrm{K})$ value should be $\pm 0^m_.3$. Introducing expressions for S and L in the relation, we find the mean error of the computed $M_v(k, n, m)$ to be $\pm 0^m_.4$, by using the mean error $\pm 0^m_.010$ adopted for k, n and m. Thus the mean

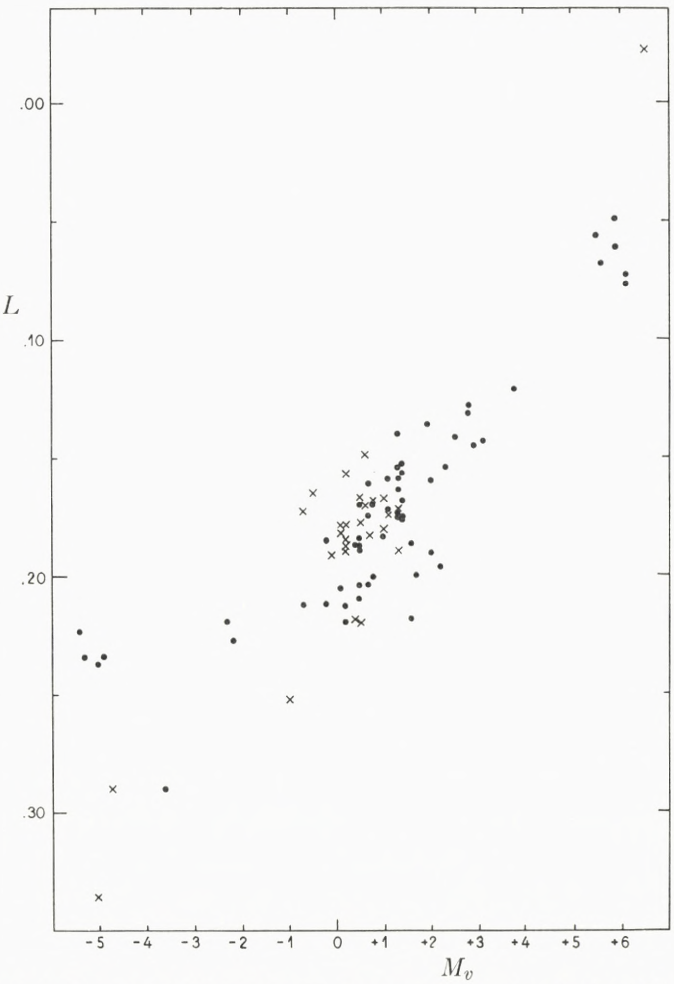

Fig. 5. The luminosity parameter L in relation to the absolute magnitude. $S < 0.80$, dots; $S > 0.80$, crosses.

square of the scatter due to the uncertainty should be 0.25, and there seems to be a cosmical scatter of $\pm 0^{m}.42$. Wilson (1959) has estimated the cosmical scatter in the relation between $M_v(\mathrm{K})$ and M_v determined from trigonometric parallaxes to be $\pm 0^{m}.3$. We have omitted a few extremely large residuals which would, of course, increase the scatter in $M_v(\mathrm{K}) - M_v(k, n, m)$. However, these residuals may be related to an effect of duplicity (cf. Section 5).

The described procedure has essentially used a transformation from the observed indices to the three independent parameters, covering the entire range of spectral types for which S is defined. Then M_v is derived from two of these parameters L and S. M_v depends primarily on L and the dependence on S has the character of a zero-point variation. This might as well be due, at least partly, to a zero-point variation of M_v(K) with spectral type, but according to Wilson (1959) such effect should be negligible. The third parameter C is assumed to represent the heavy-element-to-hydrogen ratio, but the latter may depend also on S; this question will be discussed in Sections 6 and 7.

An additional number of stars, particularly more supergiants and main-sequence stars, might permit a derivation of a least-squares relation between M_v, S, x and y from which q would be determined (the present data are too inhomogeneously

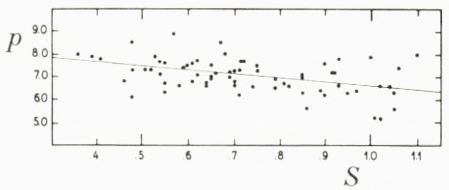

Fig. 6. The constant $p = M_v + 34.3\,L$ plotted against the spectral type parameter S.

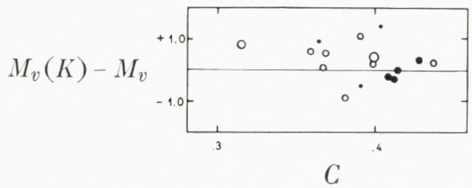

Fig. 7. Absolute magnitudes M_v (K) minus M_v determined from trigonometric parallaxes (open circles) or group parallaxes (dots), plotted against the third parameter C.

distributed relative to the scatter). Then an effect of non-linearity might also be taken into account.

When estimating q and the luminosity parameter L from M_v(K) one should consider that the reversal widths of the K- and H-lines may depend slightly on the chemical composition (cf. a suggestion by van den Bergh (1962)). This would give a correlation between residuals M_v(K) minus "true" M_v, and the third parameter. In Fig. 7, $M_v(\mathrm{K}) - M_v$ has been plotted against C for group stars and stars with M_v computed from trigonometric parallaxes larger than 0.″040. The large dots are the Hyades stars, the small dots are the remaining group stars (cf. Table 4); the large open circles are β Gem and α Boo, the small circles the remaining trigonometric parallax stars. Although the data are too limited for a conclusive evaluation, it is indicated that, if real, the effect should be small for the considered range of C. For two of the stars in the program, α Boo and α Ser, representing the smallest and largest C values respectively, $M_v(\mathrm{K}) - M_v$ differs by only $0\overset{\mathrm{m}}{.}5$.

An essential result of the preceding analysis is the agreement between $M_v(k, n, m)$ and M_v(K), which corresponds to a mean error of $\pm 0\overset{\mathrm{m}}{.}46$ for each of the two sets of absolute magnitudes. It should be mentioned that when changing q from 1.4 to 1.8, the mean square of the residuals is found to be unchanged (0.44). The agreement also justifies the application of the linear method. This is valid for the giant and the subgiant stars. It is probably not strictly valid outside this luminosity range since slight systematic residuals $M_v^{\mathrm{obs}} - M_v(k, n, m)$ are found for the supergiants and the main-sequence stars, on the average $-0\overset{\mathrm{m}}{.}4$ in the former and $+0\overset{\mathrm{m}}{.}4$ in the latter case.

By means of the expressions for x and y, L and C are found to be related approximately to the classification indices by

$$L = 0.86\,n - 0.61\,k - 0.84\,m + \text{const.}$$
$$C = 0.86\,n + 1.08\,k + 0.43\,m + \text{const.}$$

and the photometric mean error is found to be $\pm 0^{\mathrm{m}}.011$ for L and $\pm 0^{\mathrm{m}}.006$ for C.

The $M_v(k, n, m)$ values are given in Table 12 for almost all the program stars within the considered region of spectral types and luminosity classes. A few stars which have exhibited peculiarities are omitted, likewise omitted are all stars known to be spectroscopic binaries. Spectroscopic parallaxes have been computed with the aid of the absolute magnitudes together with photoelectric V magnitudes when available or m_v from the Bright Star Catalogue corrected by $-0^{\mathrm{m}}.16$ according to a comparison of m_v with V for 29 stars. These parallaxes are also listed in Table 12.

$M_v(k, n, m)$ has been tested by comparing the spectroscopic parallaxes π_s with corresponding trigonometric parallaxes π_t, published in the General Catalogue of Jenkins (1952). In order to have a material approximately homogeneous in accuracy the stars with π ranging from $0''.020$ to $0''.050$ are considered (for smaller parallaxes π_t is uncertain, for large parallaxes $\varepsilon_s = 0.2\,\pi_s$ is exceptionally high). The average mean error for the considered data of 30 stars is $\pm 0''.009$ for π_t according to the General Catalogue and $\pm 0''.006$ for π_s. The mean values for the parallaxes are $0''.0309 \pm 0''.0016$ for π_t and $0''.0297 \pm 0''.0011$ for π_s. The mean error of the scatter in $\pi_t - \pi_s$ is $\pm 0''.0096$ which is even smaller than what corresponds to the mean errors of π_t and π_s; this may be due to the selection. However, it indicates that no serious cosmical scatter should be present.

In the M_v calibration we have used the group parallax values M_v^g for the Hyades as determined from the distance moduli by Heckmann and Johnson (1956), but no particular weight has been given to these stars since the uncertainty of L is of the same order as for the remaining stars. These M_v^g values and, furthermore, the M_v determined for the Eggen group stars included in the program, are shown in Table 4

Table 4.

Comparison of $M_v(k, n, m)$ with M_v^g determined from group parallaxes.

	M_v^g	$M_v(k, n, m)$	
γ Tau	$+0^{\mathrm{m}}.68$	$+1^{\mathrm{m}}.2$	
δ Tau	+0.66	+0.1	
ε Tau	+0.54	−0.1	
θ^1 Tau	+0.80	+0.4	
$\varkappa$ Per	+1.25	+0.3	Eggen 1962
δ Ari	+1.18	+1.4	" 1962
HR 1327	+0.27	+0.3	" 1958a
ξ CrB	+1.15	+1.2	" 1960b
11 Lac	−0.50	+0.2	" 1960b
HR 5541	+1.05	+2.4	" 1959

together with $M_v(k, n, m)$. For δ Ari, HR 1327 and ξ CrB $M_v(k, n, m)$ agrees well with M_v^g, while for $\varkappa$ Per, HR 5541 and partly also 11 Lac the differences are significant. For γ Tau we have found several indications of peculiarity (cf. p. 46). Without this star the average difference $M_v^g - M_v(k, n, m)$ is $+0\overset{m}{.}5 \pm 0\overset{m}{.}2$ for the Hyades.

For the two giants β Gem and α Boo, having the largest trigonometric parallaxes, 0″.093 and 0″.090 respectively, $M_v(k, n, m)$ is found to be $+1\overset{m}{.}8$ and $+0\overset{m}{.}8$. The corresponding values derived from trigonometric parallaxes are $+1\overset{m}{.}0$ and $-0\overset{m}{.}3$. The deviations for these stars have opposite sign of the average deviation for the three Hyades stars. The M_v(K) values do also deviate from the trigonometric M_v for β Gem and α Boo, but the difference is about half that found for $M_v(k, n, m)$.

In Table 12, in which values of S, L, C, $M_v(k, n, m)$ and π_s are given, we have adopted the group parallaxes for the Hyades and the corresponding M_v values. For β Gem and α Boo we have chosen the trigonometric parallaxes and the M_v derived by utilizing these parallaxes.

4. Comparison of the Classification Parameters with the Cambridge Intensity Ratios

We shall test the classification system established in the preceding section by investigating how well other spectrophotometric quantities, i. e. those measured at the Cambridge observatory, can be represented by our classification parameters.

The comparison of the index n with the CN ratio determined by Griffin and Redman (1960) is briefly mentioned in Section 8. The G-band ratio has been measured for 61 of our stars; for 58 of them the luminosity classes are II–III, III and III–IV. A plot of $g' = 2.5 \log$ (G-band ratio) versus S indicates that these quantities are linearly correlated in the range G8 III–K3.5 III. A least-squares solution provides the relation $g' = 1.007 + 0.135\, S$, and the root mean square of the residuals is $\pm 0\overset{m}{.}020$. This scatter is accounted for by the uncertainty of g', according to the estimate made by the Cambridge astronomers. However, the residuals seem to depend somewhat on the right ascension. This effect is also found when all g' data are compared with MK spectral types. For the range G8–K3.5, II–III – III–IV, the average values $\bar{g}'$ have been computed for each MK spectral type. The G-band index, corrected for its dependence on the spectral type, is then $g' - \bar{g}'$. The average values of $g' - \bar{g}'$ for different intervals of right ascension are, 8^h–10^h: $-0\overset{m}{.}013$ (17), 10^h–12^h: $-0\overset{m}{.}012$ (17), 12^h–14^h: $+0\overset{m}{.}005$ (22), 14^h–16^h: $+0\overset{m}{.}006$ (29), 16^h–18^h: $+0\overset{m}{.}007$ (29), 18^h–20^h: $+0\overset{m}{.}004$ (23), (number of stars in parentheses). All G-band observations were carried out in 1958 April–June, and those from the period April 8–23 were rejected by the authors. If we omit all observations with $\alpha < 12^h$ from the common group of stars, we derive the relation $g' = 0.993 + 0.162$ S for data of 39 stars. The root mean square of the residuals is then $\pm 0\overset{m}{.}016$. The residuals are neither correlated with L nor with C.

In Fig. 8 the index $b = 2.5 \log$ (Mg b-line ratio) (Deeming 1960) is plotted against S. It is obvious that b depends both on spectral type and luminosity. From the mean values of S and b for the class III stars G8–K3 (large open circles in Fig. 8) we derive the linear relation $b = 0.226\,S + 0.743$; hence the quantity $b - 0.226\,S$ measures the variation perpendicular to the regression line. In Fig. 9 this quantity is plotted against $M_v(k, n, m)$. It is evident, that with the exception of a few stars separated from the majority, a good correlation exists between the two quantities. Accordingly, b should be linearly related to two of our parameters S and L and a least-squares solution provides the relation $b = 0.199\,S - 0.901\,L + 0.928$. There seems

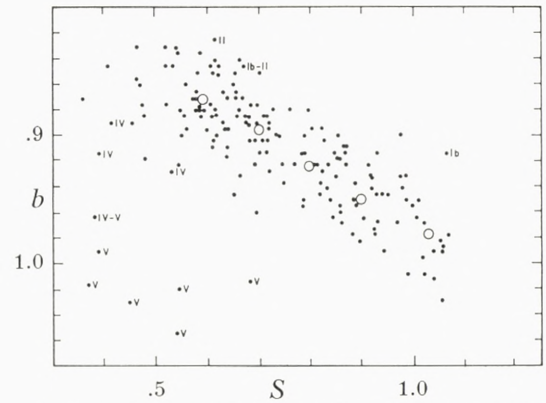

Fig. 8. The magnesium-line index b plotted against the spectral-type parameter S. Large open circles correspond to mean values of b and S for G8 III, K0 III, K1 III, K2 III and K3 III.

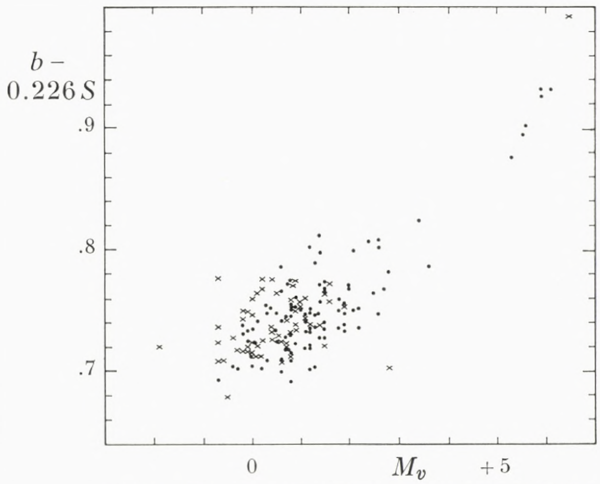

Fig. 9. The magnesium-line index corrected for the spectral-type dependence, $b - 0.226\,S$, plotted against absolute magnitude $M_V(k, n, m)$.

to be no correlation between the residuals $b_{\mathrm{obs}} - b_{\mathrm{comp}}$ and C. The root mean square is $\pm 0^{\mathrm{m}}.026$. The contribution from b_{comp} should be $\pm 0^{\mathrm{m}}.007$, while the uncertainty from b_{obs} should contribute $\pm 0^{\mathrm{m}}.009$ according to Deeming (1960). Thus, the total estimated uncertainty $\pm 0^{\mathrm{m}}.011$ does not account for the total scatter; the cosmical scatter is considerable, i.e. $\pm 0^{\mathrm{m}}.024$. An effect of non-linearity should be small. However, a plot of $b - 0.226\,S$ against $M_v(\mathrm{K})$ shows a scatter of the same order as the one found for the relation between $b - 0.226\,S$ and $M_v(k, n, m)$. In both cases the scatter can be explained approximately, if we only assume the accuracy of M_v determined from $b - 0.226\,S$ to be of the same order as the accuracy of $M_v(k, n, m)$ and $M_v(\mathrm{K})$.

The FeI ratio at 5250 A (Griffin 1961) has been measured for 43 stars in our program; 37 stars are of luminosity class III, one of class V and two of class Ib. For the class III stars the index $f = 2.5 \log$ (FeI ratio) is apparently linearly related to S, and the relation $f = 0.068\,S - 0.006$ is derived. The quantity $f - 0.068\,S$ seems to be related to L, as should be expected from the study of all FeI data measured in relation to the MK luminosity classes. Apparently there is no correlation between the remaining scatter and C, but the number of stars is too small for a quantitative discussion.

Griffin (1961) has shown that the D-line ratio is related to the spectral type with an appreciable dispersion, which is apparently not correlated with the luminosity. This is also shown in Fig. 10, where $d = 2.5 \log$ (D-line ratio) is plotted against S. The class III stars (dots without indication) are considerably scattered around the

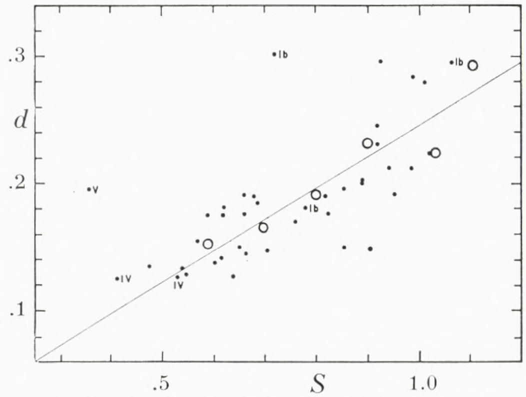

Fig. 10. The D-line index d plotted against the spectral-type parameter S.

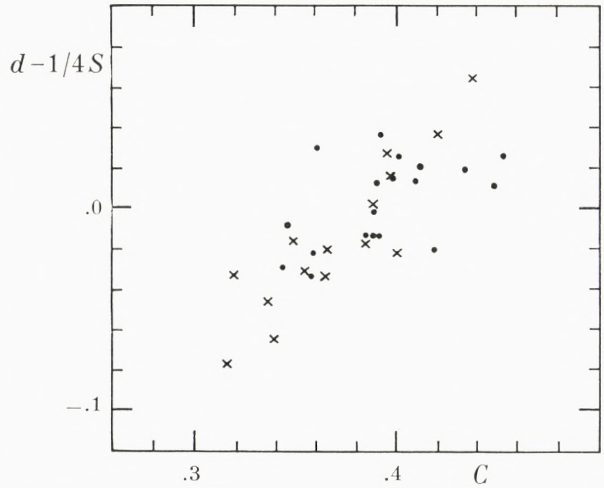

Fig. 11. The D-line index corrected for the spectral-type dependence, $d-1/4S$, plotted against the third parameter C.

mean points (large open circles), and the positions of the few stars of other luminosity classes indicate that the scatter is not due to luminosity variation among the class III stars. The limited data do not justify a substantial evaluation of the dependence of d on S, but the linear relation corresponding to the mean values should be a fair approximation for the considered range. Consequently $d-1/4\,S$ should be independent of the spectral type. In Fig. 11 this quantity is plotted against C, and it appears that most of the unknown dispersion in the D-line intensity is correlated with our third parameter (the three dot stars with largest C are noted in Table 6).

5. The Relation of the Classification Parameters to Photoelectric Colour Indices

One purpose for a classification of increased accuracy should be to provide precise indicators of intrinsic colours with a view to a determination of colour excesses. In Paper II we considered relations between the colour index $(B-V)$ and k

and g respectively. In the present section we shall make an analysis of $(B-V)$ and $(U-B)$ as well, in terms of our classification parameters, by utilizing additional colour index data. We have based our investigation primarily on data from the following sources: Johnson (1955), Morgan, Harris and Johnson (1953), Johnson and Knuckles (1957), Johnson and Morgan (1953), Roman (1955), some unpublished observations by Harris (private communication), $(B-V)$ transformed from $(P-V)$ measured by Eggen (1955), $(B-V)$ transformed from $(V-G)$ given by Stebbins and Kron (1956), and in addition some colour indices published by Wilson and Bappu (1957). Preference is given in the order in which the sources are presented here (in the remarks to Table 11 the source is indicated for each single star). Hence $(B-V)$ values are available for 80, and $(U-B)$ values for 68 of the stars for which S, L and C have been discussed. Recently Argue (1963) has published UBV data for a large number of G and K stars. About 60 stars are common in this series and the above compilation. Except for a small difference, on the average $+0^{m}.012$ in $(B-V)$ and $-0^{m}.004$ in $(U-B)$ (Argue minus the other sources), the two sets of measurements are in close agreement, the mean square of the residuals being 0.00025 for $(B-V)$ and 0.00027 for $(U-B)$. The first group of observations represents several different series all fitted to the Johnson standard system, while Argue's results are based on two homogeneous series also fitted to the standard values (Johnson 1955). In both cases a limited number of stars of the present range of spectral types has been used in the transformations. The close agreement between the two groups for a larger number of stars then demonstrates the reliability of both. We divide the mean square equally among them and adopt the mean errors $\pm 0^{m}.011$ for $(B-V)$ and $\pm 0^{m}.012$ for $(U-B)$ as representing the external accuracy of each group. This accuracy is somewhat higher than generally adopted for $(U-B)$ but of the normal order for $(B-V)$. The following relations between colour indices and classification parameters have been derived from the data of the first group. When discussing the interpretation of the residuals we have added 27 stars measured solely by Argue, after correcting the colour indices of these stars for the above systematic differences.

The colour indices should depend mainly on the spectral-type parameter, and plots of $(U-B)$ and $(B-V)$ against S indicate linear relations in the considered range. By the least-squares method we have derived the equations $(B-V) = 0.698\,S + 0.549$ from the data of 80 stars, and $(U-B) = 1.552\,S - 0.191$ from 68 stars. The root mean square of the $(B-V)$ residuals is $\pm 0^{m}.035$. The contribution from the measurement uncertainty should be $\pm 0^{m}.018$ from S, and $\pm 0^{m}.011$ from $(B-V)$. Part of the remaining scatter $\pm 0^{m}.028$ may be attributed to the other parameters L and C. Plots of the $(B-V)$ residuals against these quantities show no significant dependence on the luminosity parameter, but some correlation with the third parameter seems to be present (cf. Fig. 12). A least-squares solution involving both S and C gives the relation

$$\begin{aligned}(B-V) = {} & 0.708\ \mathrm{S} - 0.537\ C + 0.748 \\ & \pm 0.019 \quad \pm 0.109 \quad \pm 0.043\end{aligned}$$

The root mean square of the residuals is then $\pm 0^{m}.029$, while the contribution from the uncertainty of $(B-V)$, k, n and m is $\pm 0^{m}.022$. Thus the remaining scatter is reduced to $\pm 0^{m}.019$.

In the $(U-B)$ relation the scatter is $\pm 0^{m}.039$ (r.m.s.) which can be explained by the uncertainty of $(U-B)$ and S. Accordingly, the residuals show no dependence on C and L, and the above relation for $(U-B)$ is considered significant

$$(U-B) = \underset{\pm 0.028}{1.552}\, S - \underset{\pm 0.021}{0.191}$$

A few stars, quoted below in Table 6, have remarkably large positive residuals, both in $(U-B)$ and $(B-V)$; π^2UMa shows a large residual in $(B-V)$, while its colour

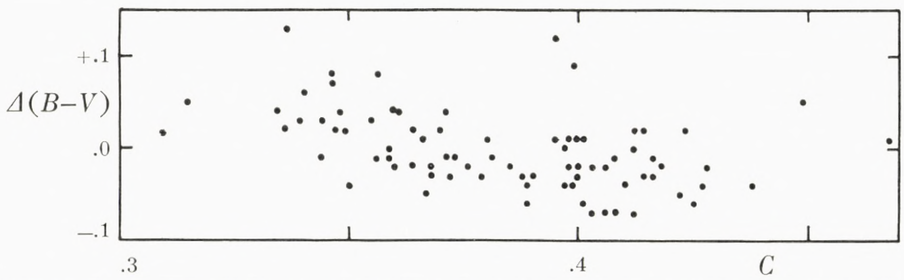

Fig. 12. Residuals $\Delta(B-V)$ from the relation of $(B-V)$ to the spectral type parameter S, plotted against the third parameter C.

index $(U-B)$ is not available. In addition HR 645 has a large negative residual $-0^{m}.12$ in $(U-B)$, but only $-0^{m}.02$ in $(B-V)$. These stars have been omitted in the above evaluation of the scatter. If they are also excluded in the least-squares computations, the first relations are found to be $(B-V) = 0.704\, S + 0.537$ and $(U-B) = 1.559\, S - 0.206$. The scatter is then $\pm 0^{m}.031$ (r.m.s.) in the $(B-V)$ relation and $\pm 0^{m}.038$ in the $(U-B)$ relation. However, since the rejection of certain observations is questionable, and since in the present case it affects the linear relations only very slightly, we adopt the above relations and the corresponding residuals in the following.

It turns out, that the residuals $\Delta(U-B)$ and $\Delta(B-V)$ are correlated, with a correlation coefficient of 0.81. The residuals are plotted in Fig. 13, as dots for the 68 stars. A least-squares solution for these stars gives the relation

$$\Delta(U-B) = 1.50\, \Delta(B-V).$$

There may be a small effect of correlation due to a slight non-linearity in the relations between S and $(U-B)$ and $(B-V)$ respectively. However, even if only stars in the range $0.6 < S < 0.9$ are considered, the correlation between $\Delta(U-B)$ and $\Delta(B-V)$ is still present. If the deviation from linearity should be real, it would influence the two residuals about equally, and would not seriously change the relation between them.

The scatter in the residuals $\Delta(U-B)$ minus $1.50\,\Delta(B-V)$ is found to be $\pm 0\overset{m}{.}032$ (r.m.s.), which is much less than would be independently caused by the uncertainty of these quantities. This may be explained by different causes. The two sets of residuals are defined as $\Delta(B-V) = (B-V) - 0.708\,S + 0.537\,C - 0.748$ and $\Delta(U-B) = (U-B) - 1.552\,S + 0.191$, where $(U-B)$ and $(B-V)$ are the observed quantities. Introducing k, n and m through the expression for S on p. 10 and the approximate expression for C on p. 15, the residuals can be expressed as $\Delta(U-B) \sim (U-B) - 1.55\,n + 2.64\,k$ and $\Delta(B-V) \sim (B-V) - 0.25\,n + 0.23\,m + 1.78\,k$. Accordingly, an error in k will affect $\Delta(U-B)$ and $\Delta(B-V)$ in the same direction, and the two contributions from k will have a ratio of about 1.5. Furthermore, $(U-B)$ and $(B-V)$ are generally measured almost simultaneously. Poor extinction correction would contribute to the mean errors of $(U-B)$ and $(B-V)$ separately, but the single errors two by two would be correlated. Both types of errors could explain the comparatively small scatter in the relation between the residuals. On the other hand it does not seem likely that the uncertainty should cause all the actual variation of the residuals. Even if the large residuals were omitted, we found a remaining scatter in $(B-V)$. When including the residuals determined for the 27 Argue stars, by means of the above relations (crosses in Fig. 13), the correlation is confirmed, but some of the latter residuals contribute to an increase of the scatter. Nevertheless, the total variation of more than $0\overset{m}{.}20$ cannot be explained by measurement uncertainties.

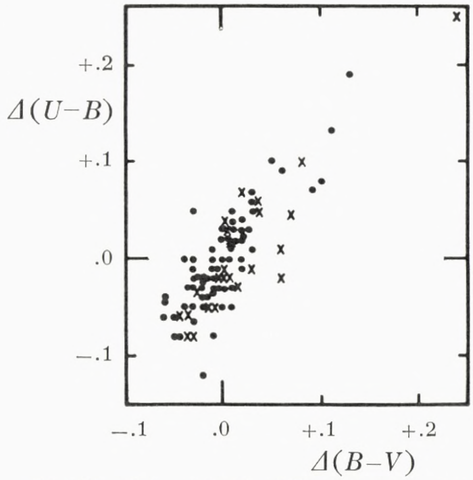

Fig. 13. The residuals $\Delta(U-B)$ from the relation of $(U-B)$ to S, plotted against residuals $\Delta(B-V)$ from the relation of $(B-V)$ to S and C.

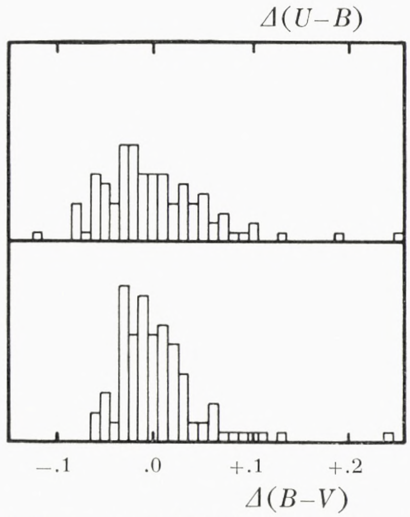

Fig. 14. Histograms of the residuals $\Delta(U-B)$ and $\Delta(B-V)$.

The detected colour effect has the characteristic of reddening according to the distribution of the colour residuals for the program stars as shown in Fig. 14. Particularly, we notice the asymmetry in the histogram of $\Delta(B-V)$, which has the smallest measurement uncertainty. Since the stars all belong to the solar vicinity, the average distance being 51 parsecs for the objects with available colour indices, the effect of interstellar absorption should be quite small. We refer particularly to an investigation by STRÖMGREN (1962a) of reddening of nearby B stars, from which it appears that the absorption within 90 parsecs is almost negligible. Therefore, it is probable that effects other than interstellar reddening are responsible for the $\Delta(B-V)$ asymmetry. We also notice that the average ratio between $\Delta(U-B)$ and $\Delta(B-V)$ is obviously larger than the value 0.73 which corresponds to the standard law of interstellar absorption.

In this context we attempt an evaluation of the influence of duplicity on the classification indices and the colour indices. It is obvious that when the components of a double-star system have equal spectral types the system will have the same indices as the single stars and no peculiarity is detected, while the spectroscopic parallax determined from $M_v(k, n, m)$ may be in error, by as much as 35 per cent if the luminosities are also equal. On the other hand, if the spectral types are somewhat different and the brightnesses of the components also differ significantly, the indices for the system will be almost the same as for the most luminous component. The strongest peculiarities will appear when the components are of different spectral types and have brightnesses of the same order.

The question of which combinations of spectral types and luminosities are really possible is related to the problem of the evolution of close binaries. Since the evolutionary paths of the components of such systems may very well be different from those of single stars because of mass exchange and related effects (cf. WOOD 1962), we cannot treat the duplicity problem in a straight-forward manner. However, it is clear that if combinations of giant stars should be frequent, such systems would give peculiarities because of the similarity of the luminosities of these stars. Therefore, we have computed the combined indices k, n, g, m, S, L, C and $(B-V)$ for different systems of class III stars by utilizing the mean values of Table 1 and the corresponding mean values of $(B-V)$ for the range G8–K5, and the values for 31 Com and HR 1327 as representing G0 III and G5 III respectively. In this manner we have considered the entire range for which our classification indices have been determined, although systems with G0 III and G5 III may be unusual. Binaries in which spectral types outside the considered range have significant influence on the composite spectrum should be detectable in the MK classification which has been carried out for the program stars. Concerning the absolute magnitudes we have adopted the MK value at G0 III for 31 Com, the $M_v(k, n, m)$ value for HR 1327, for G8 III–K3 III values computed as averages of $M_v(k, n, m)$ for each spectral type separately, and for K4 III and K5 III average values determined from M_v(K).

The results of the computation are shown in Table 5 for the S range considered throughout this paper. Combined values of S and C are given, and residuals $\Delta(B-V)$ are determined as combined $(B-V)$ minus $(B-V)$ computed by means of the relation on p. 19 from combined S and combined C. It is then remarkable that $\Delta(B-V)$

TABLE 5.
The duplicity effect.

	S	C	$\Delta(B-V)$	Δk	ΔM_v
G 0–G 8	0.389	0.422	0.05	0.042	0.9
G 0–K 0	0.482	0.433	0.06	0.043	1.6
G 0–K 1	0.544	0.441	0.09	0.047	2.2
G 0–K 2	0.625	0.454	0.11	0.059	2.6
G 0–K 3	0.727	0.466	0.14	0.085	3.1
G 0–K 4	0.776	0.480	0.20	0.095	4.1
G 0–K 5	0.806	0.493	0.28	0.121	5.4
G 5–G 8	0.511	0.401	−0.01	0.001	0.6
G 5–K 0	0.585	0.403	−0.01	0.002	0.8
G 5–K 1	0.643	0.406	0.01	0.006	1.2
G 5–K 2	0.713	0.417	0.04	0.015	1.4
G 5–K 3	0.804	0.428	0.06	0.045	1.9
G 5–K 4	0.849	0.438	0.12	0.050	2.6
G 5–K 5	0.874	0.452	0.20	0.081	3.9
G 8–K 0	0.650	0.391	−0.02	0.000	0.9
G 8–K 1	0.703	0.392	0.00	0.002	1.1
G 8–K 2	0.763	0.400	0.02	0.009	1.2
G 8–K 3	0.844	0.408	0.04	0.022	1.6
G 8–K 4	0.886	0.416	0.08	0.035	2.2
G 8–K 5	0.906	0.430	0.16	0.061	3.4
K 0–K 1	0.750	0.387	0.00	0.000	0.9
K 0–K 2	0.803	0.393	0.01	0.006	1.0
K 0–K 3	0.877	0.401	0.03	0.015	1.3
K 0–K 4	0.916	0.407	0.06	0.025	1.8
K 0–K 5	0.933	0.420	0.14	0.050	2.9
K 1–K 2	0.852	0.388	0.02	0.001	0.8
K 1–K 3	0.924	0.393	0.03	0.007	1.1
K 1–K 4	0.963	0.397	0.06	0.014	1.5
K 1–K 5	0.979	0.407	0.13	0.035	2.5
K 2–K 3	0.967	0.393	0.03	0.002	0.8
K 2–K 4	1.005	0.395	0.06	0.008	1.2
K 2–K 5	1.019	0.405	0.12	0.027	2.0
K 3–K 4	1.066	0.390	0.06	0.002	0.8
K 3–K 5	1.076	0.396	0.11	0.015	1.5

is almost exclusively positive and thus the reddening noticed above may be at least partly explained as a duplicity effect. One of our classification indices g has not been used in the classification system. It was found in Paper II (Fig. 4) that g and k are quite closely correlated with only a few stars showing deviations larger than what may be due to the measurement uncertainty. Since duplicity may affect the relation between these indices we have computed a Δk as combined k minus k determined from the relation by the mean values of k and g of Table 1 using combined g values. These Δk values are also shown in Table 5 and it is again noticeable that Δk is only positive. It should, therefore, be possible to detect the duplicity if all four classification indices k, n, g and m are measured. We notice further in Table 5 that in every case the combined C appears to be larger than the mean value 0.386 determined for all our program stars, and for the systems with one component of the type G0 III or G5 III extremely large C values are found. Finally, ΔM_v in the last column of Table 5

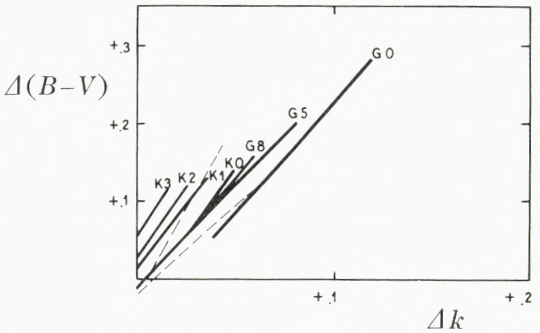

Fig. 15. Residuals $\Delta(B-V)$ for close binaries (see text) plotted against the corresponding residuals Δk.

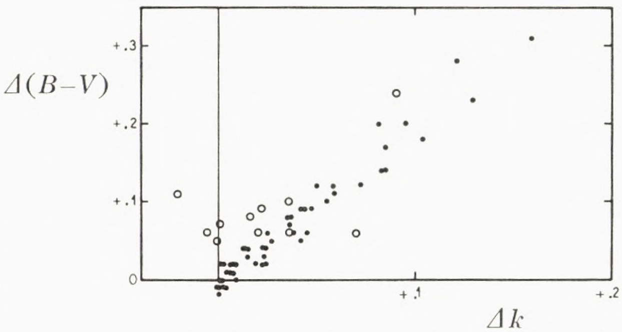

Fig. 16. Residuals $\Delta(B-V)$ for close binaries plotted as dots against corresponding residuals Δk for combined $S<0.9$ (Δm_v, Δm_v+1 and Δm_v-1). The circles correspond to observed values.

is M_v computed from combined S and combined L by means of the relation on p. 12, minus the combined M_v determined directly from the adopted average absolute magnitudes at G0 III, G5 III etc.; also ΔM_v is only positive and will have appreciable values in many cases.

In Fig. 15 we have approximated the relation between $\Delta(B-V)$ and Δk by smooth curves for the systems with one component being G0 III, G5 III, ... K3 III respectively. The G5, G8 and K0 lines are almost merging. In the above computation we have used the magnitude differences Δm_v between the components as determined from the adopted absolute magnitudes*. If we, alternately, use differences Δm_v+1 or Δm_v-1 we get lines in Fig. 15 with somewhat deviating tilts, as demonstrated for the G5, G8, K0 case by the dotted lines; largest $\Delta(B-V)/\Delta k$ corresponds to Δm_v+1.

In Table 6 we have listed the program stars with $\Delta(B-V)$ larger than $+0\overset{m}{.}04$. When attempting a distinction between interstellar and duplicity reddening one should, of course, consider the uncertainty of Δk, and an improved classification accuracy with increased sensitivity of g and k may give better separation of the two effects than is possible at present. However, for stars as 132 Tau and 73 Leo with very large Δk values and also exceptionally large values of C, $\Delta(B-V)$ is very likely due to duplicity. In fact, 73 Leo is denoted as spectroscopic binary in the General Catalogue of Stellar Radial Velocities, and for 132 Tau we have found an indication of varying spectral type (Paper I, p. 28). One additional star, HR 8485, for which

* For k, n, g and m the corresponding Δm_b has been used.

TABLE 6.
Program stars with $\Delta(B-V)$ larger than $+0^{m}.04$.

HR		S	C	$\Delta(B-V)$	Δk	$M_v(k, n, m) - M_v(\mathrm{K})$	gal. latitude	distance pc
				m		m		
163	ε And....	0.359	0.346	+0.06	+0.020	−0.1	−33°	37
1907	φ^2 Ori.....	0.481	0.346	+0.05	−0.001	+0.1	−12	20
2002	132 Tau....	0.625	0.492	+0.06	+0.070		− 2	101
2805	66 Aur....	0.858	0.374	+0.08	+0.016		+23	111
3403	π^2 UMa....	0.860	0.356	+0.06	−0.006	+0.9	+36	42
4301	α UMa...	0.658	0.449	+0.09	+0.022	+1.0	+51	16
4365	73 Leo....	0.664	0.496	+0.24	+0.091		+64	119
4377	ν UMa...	1.010	0.395	+0.13	+0.018	+1.1	+69	43
4608	o Vir.....	0.477	0.398	+0.10	+0.036	+1.4	+69	19
4928	9 Dra....	0.872	0.336	+0.11	−0.021		+51	91
5480	31 Boo....	0.621	0.454	+0.06	+0.036		+58	63
6199		0.679	0.434	+0.07	+0.001		+41	45

$(B-V)$ has not yet been measured, has $\Delta k = 0.073$ and $C = 0.474$ and is known to be a binary. α UMa has an appreciable although not very large value of Δk but C is extremely large, and this star is known to have a close visual companion ($\Delta m = 2.94$ according to KUIPER). 31 Boo is peculiar since Δk and C are large and $\Delta(U-B)$ is $-0^{m}.03$ (large $\Delta(B-V)$ and C to some extent correlated). o Vir has a high Δk value, and HR 6199 has a C value at the upper limit of the general dispersion of our data. However, for small and for moderate values of Δk a study also of the S values should increase the possibility for detecting duplicity. As apparent from Fig. 15, Δk as indicator of duplicity is most useful for the early-type systems of Table 5. In Fig. 16 the dots correspond to all the systems of Table 5 and of similar tables computed with $\Delta m_v + 1$ and $\Delta m_v - 1$ for which the combined S is smaller than 0.9. These dots are distributed fairly even along a straight line. The open circles denote the stars of Table 6 with S actually being smaller than this limit. It is then noteworthy that the stars with small Δk values all lie above the region of the dots. With due regard to the uncertainty of Δk and to the approximate character of this duplicity estimate we suggest that for 9 Dra and π^2UMa and maybe also for φ^2 Ori and HR 6199 the colour excess $\Delta(B-V)$ is due to interstellar reddening. For ε And, 66 Aur and α UMa part of the effect may be interstellar. The duplicity may also be traceable from the absolute magnitudes. We are not able to estimate the effect upon $M_v(\mathrm{K})$ but, according to Table 5, we could expect that if stars of Table 6 have large positive residuals $M_v(k, n, m) - M_v(\mathrm{K})$ this may give additional indication of duplicity. For the six stars with available residuals these are negligible for ε And and φ^2 Ori but about 1^{m} for the other stars. Generally, duplicity may contribute to the remaining scatter of the M_v relation on p. 12.

One star in Table 6, ν UMa, has $S > 0.9$, and it is then impossible to see from Δk whether the large $\Delta(B-V)$ is due to interstellar or duplicity reddening. The M_v

residual indicates duplicity, but it would be important to make a more detailed spectroscopic analysis in this case. For if this star is not a binary its position in the vicinity of the B star 33 LMi, investigated by Strömgren (1962a), is interesting. The two stars are located in a comparatively high galactic latitude with an angular distance of about 10 degrees. Their distance from the Sun is found to be almost the same, about 50 parsecs, and the mutual linear distance is then less than 10 parsecs. The stars show the same, considerably high colour excess $+0\overset{m}{.}15$ (it is reasonable to add $0\overset{m}{.}02$ to the $\Delta(B-V)$ of ν UMa according to the histogram of Fig. 14). However, if we ascribe the reddening in both cases to interstellar matter being within a small distance from the Sun, this matter should not necessarily belong to a single cloud, since two other stars in the direction between ν UMa and 33 LMi show no reddening. These stars are 46 LMi and 46 UMa with $\Delta(B-V)$ equal to $-0\overset{m}{.}06$ and $-0\overset{m}{.}03$ and distances of 29 and 62 parsecs respectively. Investigations of more stars in small angular distances from 33 LMi and ν UMa are desirable. However, if these stars are not binaries their location relative to the other stars together with the $\Delta(B-V)$ values, indicates a steep increase of the interstellar absorption. If ν UMa should be a binary as is possible according to the above analysis*, the steep increase of $\Delta(B-V)$ for 33 LMi is still remarkable.

6. The Third Parameter

On the basis of the classification indices k, n and m we have derived three independent empirical parameters S, L and C, where S and L are closely related to the spectral type and the absolute visual magnitude. Concerning the third parameter C we are at present not able to relate it quantitatively to any physical or chemical quantity of the stars, but we shall assume that C varies mainly with the chemical composition.

In the high-dispersion analysis by M. and B. Schwarzschild, Searle and Meltzer (1957) the abundance estimate has been concentrated on the high-velocity giant φ^2 Ori, and it was concluded that this star deviates from the low-velocity giants in having a lower metal abundance by approximately a factor of 4 and a lower abundance of the oxygen group by a similar, but possibly somewhat smaller, factor. The C value of this star is 0.346 and for the low-velocity giants investigated in the analysis and measured also in our program, the average value of C is found to be 0.385. Thus a small value of C should correspond to a comparatively small metal content. Furthermore, according to the abundance analysis similar deviations as for φ^2 Ori, though of a smaller amount, appear to hold for the high-velocity giant stars 14 And and α Boo. However, for the latter star we find a considerably smaller value of C, 0.315, than for φ^2 Ori, but this discrepancy is possibly not real. The remark on the abundance for 14 And and α Boo is made from a judgement of the spectral deviation of these stars from low-velocity giants relative to the deviations for φ^2 Ori. In fact, the

* Recent measurements in other spectral regions support this assumption (note added in proof).

TABLE 7.
Mean values of the third parameter for different spectral types.

MK	$\bar{C}$	number of stars
G 8 III	.386	36
K 0 III	.386	48
K 1 III	.383	13
K 2 III	.387	29
K 3 III	.396	20

iron-to-hydrogen ratio should be directly related to the average equivalent width W of weak FeI lines. Among the 16 high- and low-velocity stars studied, nine stars are common to our program, and for eight of them the relation $W = 0.051\ S + 0.177\ C - 0.003$ is derived. The scatter of the W residuals is ± 0.006. The additional common star γ Tau is known to exhibit peculiarities and it also shows a large W residual, but there is no discrepancy for φ^2 Ori, 14 And and α Boo.

According to the estimate for φ^2 Ori the abundance factor corresponding to the dispersion of C for our program stars should be of the same order as the one found by STRÖMGREN (1963a) for the late F- and early G-type main-sequence field stars.

The average values of C for different MK spectral types of luminosity class III are given in Table 7. It appears, as we should expect, that $\bar{C}$ is nearly constant in the whole range, and only a small increase from K2 to K3 is noticed. The dispersion in C is considerably larger than what corresponds to the uncertainty of this parameter, and it is of the same order for different spectral types as shown in Fig. 17. To some extent differences in abundance and in temperature may similarly influence the

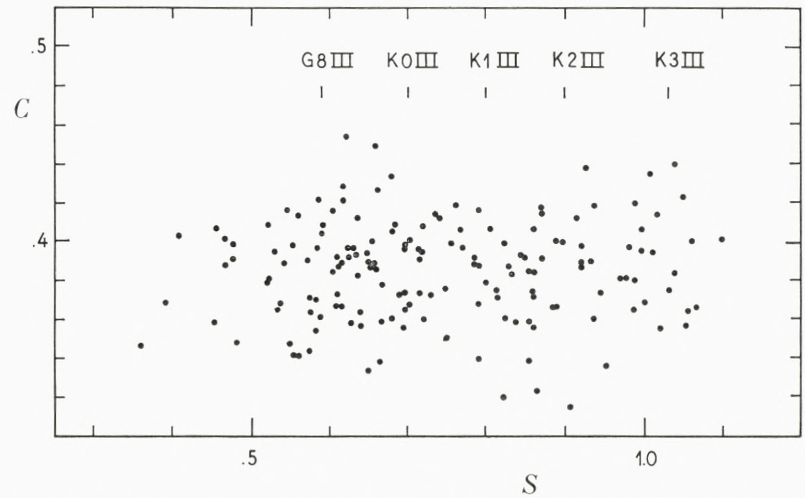

Fig. 17. The third parameter C plotted against the spectral-type parameter S; the stars with the largest C values are excluded here and in the computations for Table 7 (cf. p. 24–25).

individual spectral type estimates (cf. PAGEL 1962). The parameter S is defined by means of average spectral types and should be expected to be mainly a temperature parameter and C mainly an abundance parameter. However, as already emphasized, the correct relation between the heavy-element-to-hydrogen ratio and C may also involve the parameter S.

In this connection we shall consider again the colour index relations from the preceding section. From the $(U-B)$ and $(B-V)$ relations it appears, that C increases with decreasing intensity of the V region relative to the intensities of the U and B regions. If we assume that S and C are mainly parameters for the temperature and heavy element abundance respectively, we may interpret the variation of C with V so that for a certain value of S, the metallic line effect on the U and B region is so heavy that a further increase of the line absorption is small for these regions compared with the effect on the V region. This would again suggest that the increase of C with m (cf. p. 15) is due to a decrease of I (4970) relative to I (4050) and I (4510), rather than to a decrease of I (4050) relative to I (4510) and I (4970) as for the F stars.

The relation of k to C (p. 15) is also quite remarkable. If k is a measure of the K-line intensity an increase of k would correspond to an increase of I (3920) relative to I (4070) (cf. the definition of k in Paper I), and thus to a decrease of the K-line intensity. Consequently the composition effect on the K-line would be reversed. However, it may be questionable whether the k-index is primarily a measure of the K-line intensity. In this connection we recall the results of Paper II (p. 19), i.e. that k is well correlated with the break around the K-line, as measured by WESTERLUND (1953), while there is considerable scatter in the relation between k and the K-line intensity determined by this author. VAN DEN BERGH (1963) has recently shown that for main-sequence F and G stars the quantity Δ, measuring a discontinuity at 4000 A, is a sensitive metal-abundance equivalent. Since Δ covers almost the same spectral region as k it may well be that the Δ effect is of higher influence on this classification index than the K-line. In this connection we should recall that the spectral region of the short wave length filter (peak around 3910–3920 A) has a relatively large width of about 120 A and is not quite symmetrical; the "wing" transmission is higher on the ultraviolet side than on the red side of the maximum. This asymmetry is partly reduced by the decrease of the photocathode spectral response and the field lens transmission against shorter wave lengths. The ultraviolet part has turned out to affect the k-index considerably; attempts with a noviol filter (CHANCE OY 10) in addition to the 3920 filter gave an index of some 24 per cent smaller sensitivity than the first k although the half width was only slightly reduced. The two indices are closely correlated within the measurement uncertainty. The difference is a kind of colour effect but may be ascribed to a difference in the Δ effect as well. Recent tests utilizing a new filter for the short wave length region with peak at 3910 A and half width 60 A have shown that the corresponding k-index is about 1.7 times as sensitive as the present k. Some additional scatter is present, but a preliminary estimate indicates that the new k can be evaluated satisfactorily in terms of S, L and C. This has not appeared to be possible if the short wave length region is centred at 3850 A (CN-absorption) or at 3930 A (K-line).

However, for a given colour index or spectral type, Δ is smaller for metal-deficient main-sequence stars than for stars of normal metal abundance, and if k chiefly measures the same effect as Δ, and if C is primarily an abundance parameter, the relation of k to C would indicate that for the giant stars the discontinuity dependence on the metal deficiency is reversed. This would be similar to what is found from the UBV relations.

Quite a few giant stars are listed in Table 1 of van den Bergh's paper and only 9 stars are common with our Table 12 (two others are spectroscopic binaries). For these stars, Δ is found to be closely related to S through a least-squares linear relation $\Delta = 0.456\,S + 0.435$. The residuals range from -0.04 to $+0.03$ and the C values from 0.315 to 0.414. When the stars with the four smallest and the four largest C values are considered, the average values for the two groups are

$\bar{C}$	$\overline{\Delta_o - \Delta_c}$
0.336	+0.02
0.405	−0.01

indicating a vaiiation of Δ with C which is in agreement with what is suggested from the k variation.

It should be emphasized that these interpretations are based on the assumption that the dependence of S on the abundance and of C on the temperature, is small, and if this condition is not fulfilled the above picture will, of course, be changed.

Also we have considered the relation of our classification parameters to the long wave length colour index $(R-I)$ as determined by Stebbins and Kron (1956), although only limited data are available. By comparing $(R-I)$ and S for 28 of our program stars, it appears that this colour index depends linearly on S, and the relation $(R-I) = 0.384\,S - 0.094$ is derived. The scatter of the residuals is $\pm 0^{\mathrm{m}}.029$, which is considerably more than is expected from the uncertainty of $(R-I)$ and S. Plots of the residuals against L and C indicate some correlation with C, and the following relation is then derived:

$$\begin{aligned}(R-I) = {} & 0.405\,S - 0.335\,C + 0.024 \\ & \pm 0.029 \quad \pm 0.181 \quad \pm 0.069\end{aligned}$$

We notice that $(R-I)$ depends relatively on S and C in a similar manner as $(B-V)$; almost the same ratio of the C and S coefficients are found in the two cases, -0.76 for $(B-V)$ and -0.83 for $(R-I)$. However, the uncertainty of the ratios, mainly in the latter case, is quite large. It may be expected that $(R-I)$ is influenced considerably less by the abundance effect than $(B-V)$ and therefore may provide a more pure colour equivalent. If $(R-I)$ should not depend on the abundance at all, this would imply that S and C could be expressed as $S \sim S' + 0.8\,C'$ and $C \sim C' - 0.8\,S'$, where S' and C' are pure equivalents for the temperature and abundance respectively. However, this strong correction from (S, C) to (S', C') does not seem likely. In fact, through the analysis in the next section, it is found that the dependence of S and C on abundance and temperature respectively, must be much weaker.

Calibration through high-dispersion analysis of selected stars is clearly desirable but may be uncertain due to complications in the theory of stellar atmospheres for giant stars. Perhaps the relations between S and C and the stellar atmospheric parameters could be established most precisely in an empirical way by extending the classification measurements to stars in groups and clusters covering the entire range of giant spectral types.

The situation may be even more complicated if we consider a result of Strömgren (1963a) for the main-sequence stars. The metallic-line index m_1 exhibits a large dispersion for the A stars which cannot be due to abundance differences, and since A stars evolve into the giant branch it might be that effects other than the abundance do influence m. Our third parameter depends only partly on this index, but it is found in the next section that C has a considerable variation also for the giants evolved from the A stars. Thus groups and clusters of intermediate as well as high ages should be observed in order to elucidate the significance of the third parameter.

The direct relation between $(U-B)$ and $(B-V)$ is $(U-B) = 2.035\ (B-V) - 1.212$, computed from data of the 68 stars. The residuals u of this relation are plotted against the third parameter in Fig. 18. An increase of u with C is clearly apparent in this figure. This is not surprising since $(-u)$ would be a measure of the "ultraviolet excess" and C should increase with increasing metal content. However, according to the above discussion, the decrease of u (related to a decrease of C) should be due to a decrease of V relative to U and B, rather than to a decrease of U relative to B and V. Furthermore, the scatter in Fig. 18 is quite large. The dots surrounded by circles

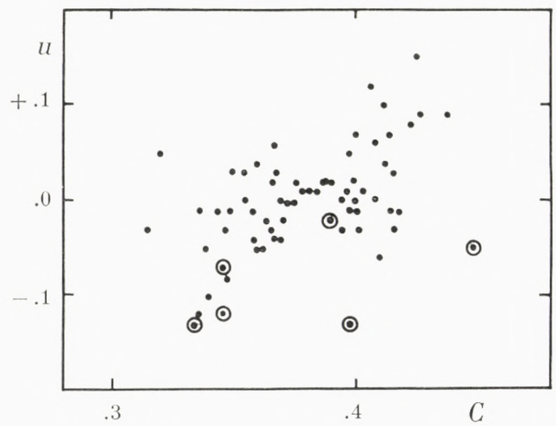

Fig. 18. Residuals u from the relation between $(U-B)$ and $(B-V)$, plotted against the third parameter C.

correspond to 6 stars with extremely large positive $\Delta(U-B)$ and $\Delta(B-V)$, and thus the effects of the third parameter and the reddening (interstellar or duplicity) will interfere in the (u, C) diagram. Furthermore, substituting in the expression $u = (U-B)_{obs} - (U-B)_{comp} = (U-B) - 2.035\ (B-V) + 1.212$, the linear relations for $(U-B)$ and $(B-V)$ as functions of S and C found above, we get $u = 0.111\ S + 1.093\ C - 0.501$, which shows, that u should depend also on S.

Although we are not able at present to relate C correctly to the stellar atmospheric parameters, we consider this quantity as an empirical characteristic similar to the weak-line, strong-line, weak-CN, and 4150 features used earlier. We shall examine how it is distributed among the program stars, relative to the distribution of the two other astrophysical parameters and to the space velocity and its galactic components. Instead of considering the independent parameters S and L, we use $(B-V)$ as directly measured or as derived from S and C, and M_v as given in Table 12. In this way we investigate the distribution of the stars in the colour-magnitude diagram relative to well known cluster sequences.

7. The Classification Parameters and their Relation to Space Velocities for the Program Stars

During recent years it has become increasingly clear that the kinematic situation for the stars in the solar vicinity is complex and rich in systematic features related to the physical characteristics of the stars. Delhaye (1948) found an indication of a preferential motion of A and F stars in the direction of the Ursa Major cluster velocity. Parenago (1950) determined the solar velocity and the apex coordinates, the vertex direction, and the axes of the velocity ellipsoid for different spectral types of the main sequence and concluded that the O–F stars form a flattened subsystem, and that the G–M stars form an intermediate subsystem. Through extensive investigations of space velocities and partly on the basis of spectroscopic population criteria, Vyssotsky and collaborators (cf. Vyssotsky 1957) have separated the neighbouring stars into two groups. The one exhibits a small dispersion of peculiar velocities and a vertex deviation, and the other, including stars with a larger dispersion in the velocities, shows no vertex deviation. Investigations of the vertex deviation have been presented in a number of papers (Strömberg 1946, Hins and Blaauw 1948, Delhaye 1952, Alexander 1958, van Rijn 1960). The deviation seems to characterize the motion of the stars that move close to the galactic plane and may be detected for stars as faint as 13^m (Vyssotsky 1957). It has been interpreted as being due to streaming, perturbations or to the evolution of spiral arms, and by Lindblad (1958) in terms of dispersion orbits, i.e. orbits along which an association of stars tends to disperse in the central field of the Galaxy. Lindblad suggested that the young stars showing the deviation may have originated in a vast cloud, which, according to Oort (1958), may be part of a spiral arm.

The spectroscopic population criteria used by VYSSOTSKY were the strength of the G-band and the hydrogen lines, the 4227-line of neutral calcium, and for dwarf M stars emission lines of hydrogen or ionized calcium. Part of the data was subdivided according to the weak-line and strong-line criteria of ROMAN (1952). BLAAUW (1958) has carried further the weak-line and strong-line differentiation for selected spectral types, and has found that the strong-line G8 III giants exhibit two concentrations of the projected velocity vectors in the galactic plane, similar to the concentrations for A0–A9 stars demonstrated by DELHAYE (1948, op. cit.). The K0 III strong-line giants show a concentration between the two G8 III groups. This picture indicates kinematic similarities between stellar types having approximately equal ages according to current theories of stellar evolution.

WOOLLEY and EGGEN (1958) have classified the stars within 20 parsecs of the Sun according to the closeness with which they approach the galactic centre. The colour-luminosity arrays of the classes indicate a decreasing pericentric distance with age, from the youngest class with an array similar to the Pleiades and nearly circular orbits, to the oldest M 67-like stars penetrating to regions within 3.5 kpc from the centre. From galactic velocity components determined by EGGEN (1962), EGGEN, LYNDEN-BELL and SANDAGE (1962) have computed orbital eccentricities and angular momenta for 221 dwarf stars and have shown that correlations exist between the observed ultraviolet excess for these stars, recognized as measuring the metal abundance, and the eccentricity, the angular momentum and the velocity component perpendicular to the plane, respectively. A correlation between the Z velocity component and the metal-to-hydrogen ratio is also found by WALLERSTEIN (1962). YOSS (1962) has indicated evidence of a correlation between the CN-anomaly for giant stars and the dispersion of the three velocity components.

In his series of stellar group investigations EGGEN (1958b) found indications of non-random distributions of the galactic plane velocity components for G1–K5 main-sequence stars when the data are subdivided according to the component perpendicular to the plane. EGGEN (1960a) further studied the components in the plane of AV stars and has detected evidence for the existence of subgroups, whose stars have a common value of the component in the direction of rotation. According to KING (1961) such stars may have originated simultaneously in a region with the same distance from the centre as our present distance; the age should be equal to the epicycle period $\pi/\sqrt{B(B-A)}$, where A and B are the Oort constants.

Such kinematic features are related to comparatively young stars and indicate a common origin; they should be expected among the older stars in the solar vicinity only under special circumstances. The high-velocity stars may be an example of this. VAN WIJK (1956) has discussed the origin of the disk high-velocity stars and has shown that these stars have probably been formed close to the perigalacticon of their orbits, a smaller fraction of them close to the apogalacticon.

The relation between kinematic and age properties of field stars are more prominent when observational methods of increased accuracy are used. We refer to results from high-dispersion quantitative abundance analysis of selected stars by Greenstein and collaborators, and particularly to the very extended investigations of nearby B, A and F stars made by Strömgren and associates, using narrow-band and intermediate-band techniques. Strömgren (1962b) obtained data for about 1200 A2–G2 stars brighter than 6^m5 in the u, v, b, y four-colour system and calibrated the measurements in terms of metal abundance and age. It was found that the average speed relative to the local standard of rest, is correlated with the age and the chemical composition respectively, for stars of small and intermediate velocities. For the region F4–G2 and luminosity classes IV and V, stars as old as the galactic cluster M 67 definitely have a higher average velocity than younger stars, and there is further an increase in the average velocity with decreasing metal content. On the basis of the same data of A and F stars, supplemented with results for B8–B9 stars (Crawford 1963) the distribution of the velocity components in the galactic plane was studied by Strömgren (1963a) for different age groups. Certain features for stars of ages between 200 and 600 million years, already noticeable in earlier similar investigations by Delhaye (1948), are clearly outstanding and are presumably related to star formation in spiral arms.

When attempting similar studies for the giant and subgiant region we encounter serious difficulties in the age determination due to the funnel effect, and, to some extent, to the overlapping of evolutionary sequences corresponding to quite different ages. However, kinematic features similar to the afore-mentioned features for evolved main-sequence stars might be found also for stars in the giant region and might support a segregation of age groups there.

In comparison with the A and F main-sequence observations described above, the present data are limited by a smaller accuracy of the location in the HR diagram and of the tangential velocity, and the third parameter needs further interpretation and calibration. Furthermore, the comparatively small number of stars should make a detection of age groups uncertain. The following, therefore, gives only a provisional kinematic picture for stars of the solar neighbourhood in the giant and subgiant regions.

The space velocities have been computed utilizing the parallaxes determined in Section 3, the radial velocities from the General Catalogue by R. E. Wilson (1953) and proper motions provided in the following way. For about one hundred stars, the proper motions listed in the FK4 catalogue or its supplement have been used. For about fifty of the remaining stars, the N30 proper motions have been corrected to the FK3 system utilizing the tables of H. R. Morgan (1953) and then to the FK4 system by means of tables of the differences FK4 minus FK3 (provided in advance of publication by W. Gliese). For the remaining stars (about thirty) the Boss GC proper motions have been corrected to the FK3 system by means of the tables of Kopff (1939) and then from FK3 to FK4. The space velocity and its galactic components determined in the 1958 system are reduced to the local standard of rest utilizing the standard solar motion (note, p. 54). The total velocity Q and the components X,

Y and Z are given in Table 12. Z is perpendicular to the galactic plane, positive towards the north pole, Y is the component in the anticentre direction, and X in the direction of the rotation (unit km/sec).

In considering the third parameter in relation to the total space velocity, we have divided the data into three groups according to C with an equal number of stars in each group. The average space velocities for these groups are shown in Table 8, where the first section comprises only stars of small and intermediate velocities, while the last section includes also the high-velocity stars. We have chosen the limit at the velocity 63 km/sec (where the asymmetry in the distribution of the plane components begins).

TABLE 8.
Average space velocities in relation to the third parameter.

C	$Q < 63$		all Q values included	
	$\bar{Q}$	number of stars	$\bar{Q}$	number of stars
0.315 – 0.373	30	42	43	54
0.374 – 0.397	25	49	28	53
0.397 – 0.468	26	53	27	54

It is seen that there is practically no change in $\bar{Q}$ for C larger than 0.375, while $\bar{Q}$ increases when C decreases below this limit. This is quite clear when the high-velocity stars are included, but also if only stars with $Q < 63$ are considered the effect should be real since the mean error of each of the $\bar{Q}$ values is about 1–2 km/sec.

If we interpret C as a measure of the chemical composition the results for the high-velocity stars agree with what is generally expected from the properties of these stars. For the low- and intermediate-velocity stars the tendency is in agreement with STRÖMGREN's results for the F4–G2 main-sequence stars, but the effect is more pronounced in the latter case. In his similar division STRÖMGREN (1962b) finds $\bar{Q}$ to be 28.5, 32 and 36.5 km/sec respectively. We shall return to this point later.

Fig. 19 shows C histograms for all the program stars and for the weak-line and strong-line stars included in the program. The first histogram indicates that the distribution of C is not quite symmetrical. From the weak-line and strong-line histograms some correlation appears in the sense that weak lines correspond to small C values and strong lines to large C values. As shown in Table 9 the average values of C for the two groups are clearly different. The difference is not very pronounced, but our data include somewhat less than half of the G and K stars for which the weak-line and strong-line characteristics have actually been indicated. These are only G8–K1 stars while the present discussion comprises the types from G5 to K3.5. The difference in C for the cyanogen criteria also given in Table 9, is quite significant.

From the plot of the velocity component Z against C in Fig. 20 we see a marked difference in the dispersion of Z for small and large C values. Again we refer to what

Table 9.
Mean values of the third parameter for Roman's spectroscopic groups.

	$\bar{C}$	number of stars
weak-line stars	0.378	55
strong-line stars	0.394	43
weak-CN stars	0.352	9
"4150" stars	0.406	11

has been found for the A2–G2 main-sequence stars (Strömgren 1963a, p. 34), that among the 1217 stars measured in the u, v, b, y photometry the few stars having velocity components at right angles to the galactic plane larger than 35 km/sec all have a comparatively small metal content (smaller than 0.3 times that of the Hyades stars).

In the investigations of dwarf and subdwarf stars (Wallerstein 1962, Eggen, Lynden-Bell and Sandage 1962), a steady increase of the dispersion σ_Z with decreasing metal content or increasing ultraviolet excess has been found. Fig. 20 and Table 10, in which the mean values $|\bar{Z}|$ and the σ_Z values have been computed for successive intervals of C, indicate that for the present data there is rather an approximately constant dispersion in Z for C larger than a limit around $C = 0.38$, and a steep increase in the dispersion below this limit.

In view of this result and furthermore with the indications of the average speed $\bar{Q}$ in Table 8 and the histograms of Fig. 19 in mind, we proceed in a manner similar to

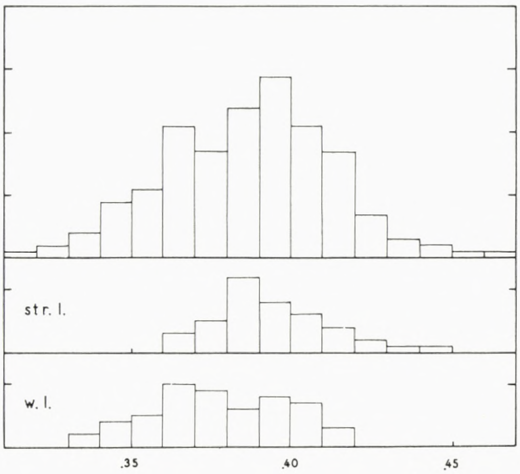

Fig. 19. Histogram of the third parameter C for all stars and for the strong-line and weak-line stars separately (binaries included).

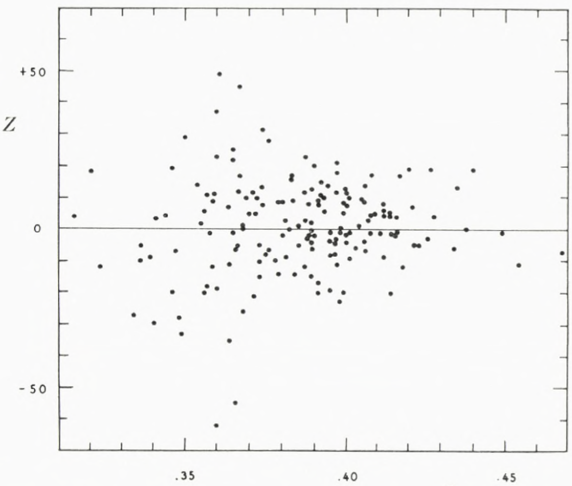

Fig. 20. The space-velocity component Z perpendicular to the galactic plane, plotted against the third parameter C.

TABLE 10.

Mean values of the numerical Z velocity component (perpendicular to the galactic plane) and values of the Z dispersion for different intervals of the third parameter.

C	$\lvert \bar{Z} \rvert$	σ_Z	number of stars
0.340–0.349	16	20	7
0.350–0.359	12	15	11
0.360–0.369	22	29	21
0.370–0.379	12	15	17
0.380–0.389	8	10	24
0.390–0.399	10	13	29
0.400–0.409	7	9	20
0.410–0.419	7	10	11
0.420–0.429	9	12	7

VYSSOTSKY and collaborators; we divide the program stars into two main groups A and B according to the value of the third parameter and tentatively set the limit at $C = 0.38$. When considering the colour-magnitude diagrams of the two groups we notice a striking difference. Group A, including the stars with C larger than 0.380, populates the diagram above the level of the sequence of the galactic cluster M 11, while group B contains very few stars above the Hyades sequence. We interpret the difference so that the group A contains a number of young giants having evolved from A-type stars, while group B consists mainly of stars of small masses evolved from the F and early G types of the main sequence. In other words, below a certain limit of our third parameter no young giants are found in our program.

If we consider only the region above the Hyades level the number of stars for successive intervals of C are 2 for the range 0.360–0.369, 5 for 0.370–0.379, 14 for 0.380–0.389, 12 for 0.390–0.399, 11 for 0.400–0.409 and 8 for 0.410–0.419. The smallest value of C is 0.366. Remembering the photometric uncertainty it seems reasonable to adopt the limit at 0.380. In group A we then include stars with $C > 0.380$, and in addition five of the seven stars above the Hyades level with $C < 0.380$. These five stars all have $|Z|$ values below the dispersion $\sigma_Z = 10$ km/sec found for all stars with $C > 0.380$. The remaining stars with $C < 0.380$ are included in group B. With the duplicity effect in mind, we exclude all stars with $C > 0.430$ from the following discussion.

The two colour-magnitude diagrams are then shown in Fig. 21. We have constructed these diagrams using $(B - V)$ from the sources quoted in Section 5, or computed from S and C by means of the equation on p. 19. The giant sequences of some galactic clusters are shown in the diagram. The M 67 and NGC 188 curves are drawn through the normal points corrected for reddening, published by SANDAGE (1962). The Hyades line is based on the data used in Table 12 (reference p. 15), and the NGC 752 and M 11 sequences are reproduced from an earlier paper of SANDAGE (1957).

Group B contains about 60 stars, while group A consists of a little less than 100 stars. Vyssotsky's groups A and B have about equal numbers of stars, but refer only to G5–K1 stars of luminosity classes II–III, III and III–IV, while our investigation extends as far as K3.5 and also includes a few stars of luminosity class IV (and the separation criterion is not determined quantitatively in Vyssotsky's work). Our data are probably subject to a selection effect since the absolute brightest stars, being so numerous in group A, have been favoured in our program ($m_v < 5.5$, mainly),

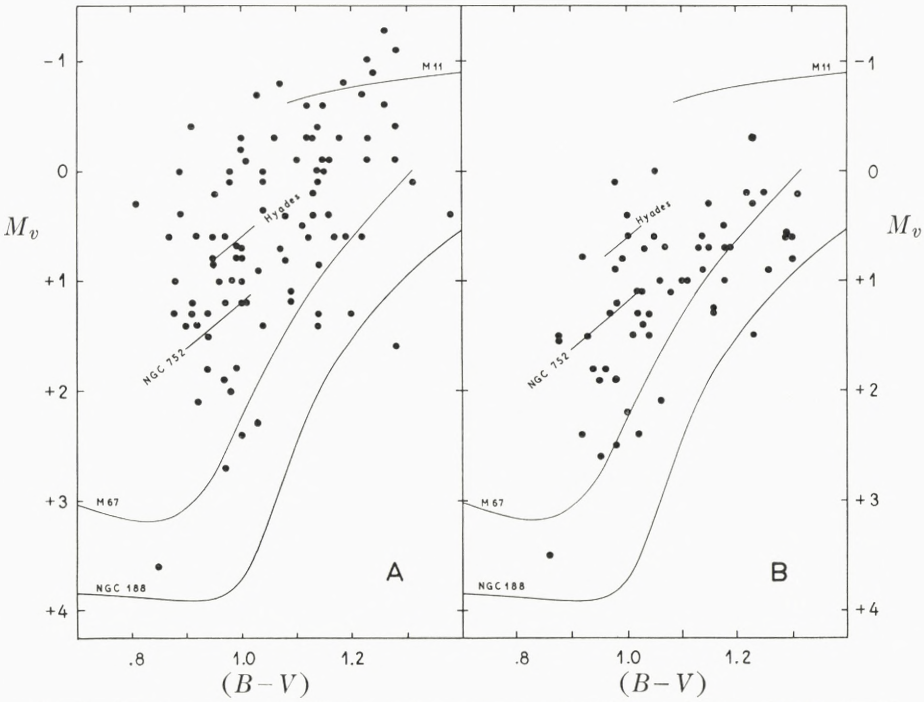

Fig. 21. Colour-magnitude diagrams for group A ($C > 0.38$) and group B ($C < 0.38$).

and the colour-magnitude diagram of field stars in the giant region should be most abundantly populated at about $M_v = 1.0$ according to Halliday (1955) and Sandage (1957). From evolutionary considerations the latter author estimated that about 70 per cent of the present K0–K2 giants were originally F2–F7 dwarfs, while most of the remaining 30 per cent were originally A0–A5 main-sequence stars. The velocity dispersions, as shown by Sandage (1957) by means of data compiled by Parenago (1950), indicate that the majority of the K0–K2 giants evolved from the F dwarfs.

However, this was mainly an age and mass division, while our division pertains to the chemical composition, and group A apparently includes also a number of stars evolved from the F- and G-type main sequence. According to the location in the colour-magnitude diagrams some stars of groups A and B have approximately the same age, as high as M 67 or even more, and have quite different C values, ranging from 0.35 to 0.40. A single star, HR 4181, lying below the NGC 188 sequence in the

$(B-V, M_v)$ diagram of group A, has the C value 0.383, which is close to the limit between the two groups. This picture is an agreement with what has been found by Strömgren (1963a) and by other investigators, that low as well as high metal content is found among very old stars, indicating that heavy element synthesis has occurred at an early state of the evolution of the Galaxy.

Returning to Table 9 and the question of the average speed in relation to C we recall that Strömgren's subdivision is concerned with stars from F4 to G2 of the main sequence. The number of stars in our data, definitely known as evolved from this region, is far too small for a conclusive discussion of the relation between C and Q, and, furthermore, some of these stars have comparatively large Z values. Table 8 presents chiefly a distinction between group A and group B with $\bar{Q} = 28$ and $\bar{Q} = 43$ respectively when the high-velocity stars are included. If for $Q < 63$ km/sec only the stars below the Hyades level are considered, the $\bar{Q}$ values are practically unchanged. For the total number (90) of these stars $\bar{Q}$ is 28 km/sec while Strömgren finds $\bar{Q} = 32$ for the F4–G2 stars. This indicates that a number of the considered stars have evolved from late A and early F stars as should be expected.

The above differentiation of two main groups A and B presents some uncertainty in the interpretation of our third parameter. For the old stars, say older than NGC 752, C would be a measure of the chemical composition with a possible correction depending on S taken into account. However, for the young giants evolved from A-type stars, no variation of the initial abundance (cf. Strömgren 1963a) should be expected, but for these stars the C variation is still quite significant. If we consider only the stars above the Hyades level in the group A colour-magnitude diagram we find a dispersion of C of $\pm 0\overset{\text{m}}{.}014$ (about 50 stars), which is considerably greater than the photometric mean error of C (cf. p. 15). This should be compared with the scatter of Δm_1 determined by Strömgren (1963a). This quantity has a comparatively small range in the region of early F stars and the dispersion increases in the direction of the later F stars due to increasing dispersion in the chemical composition. For A stars the variation of Δm_1 for a certain spectral type is again very strong and here the interpretation is not quite clear. It would, furthermore, be a matter for discussion whether the effect suggested for explaining the difference in dispersion between early F and late A-type stars, viz. the existence or non-existence of a deep outer convection zone (Strömgren 1963a), is also found in the giant region.

In this connection we notice that the four Hyades giants which belong to group A have practically equal C values for γ Tau, δ Tau and ε Tau, on the average 0.411, while for θ^1 Tau C is 0.428. In the estimate of the dispersion for the young giants we have omitted stars with C larger than 0.430 with the duplicity effect in mind (cf. p. 24). It may be that part of the dispersion is due to the duplicity systematically causing large C values; the θ^1 Tau deviation in C from the other Hyades might be explained in this way.

In order to elucidate the significance of the variation of the third parameter for young giants, an increased classification accuracy would be desirable, and, in addition, extended observations of other spectrophotometric equivalents should be made. As mentioned on p. 18 part of the variation of the D-line intensity seems to be correlated with C. However, measurements of the D-line ratio are available for only few giants above the Hyades level. It is found that two of these stars with negative values of $d - 1/4\ S$, on the average -0.016, have a mean value of C of 0.385, while three stars with positive $d - 1/4\ S$, average value $+0.041$, have $C = 0.435$. This indicates a real scatter also for $d - 1/4\ S$ for the young giants.

Furthermore, it should be emphasized that the stars above the Hyades level may not be termed young giants in every case. Some of the stars in this region, pre-

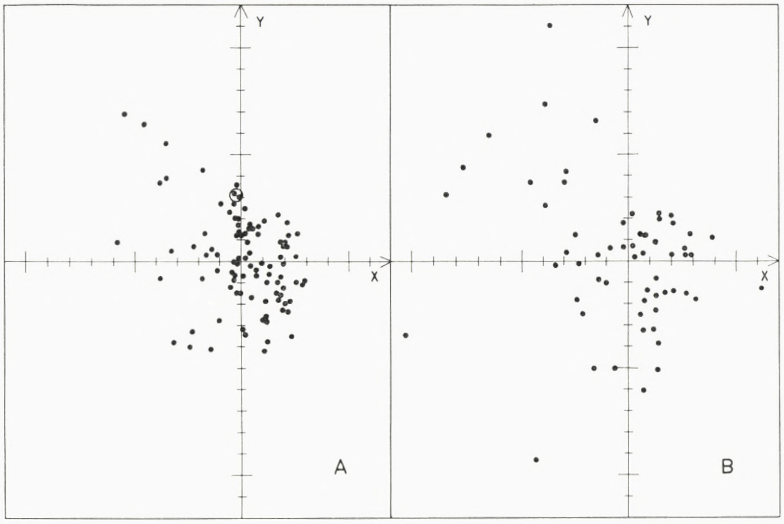

Fig. 22. Velocity components in the galactic plane for stars of group A ($C > 0.38$) and group B ($C < 0.38$). X-axis, rotational direction. Y-axis, anticentre direction. Unit 10 km/sec.

sumably a small fraction, may be old stars in late giant phases, passing from the right to the left side of the HR diagram, and for these stars the third parameter could be related to the abundance. Giants of the types belonging to intermediate-age clusters such as NGC 2158 (Arp 1962) would also partly appear above the Hyades level.

However, our interpretation of the absolute bright members of group A as being mainly evolved A-type stars is supported by the (X, Y) distribution in Fig. 22. Group A (including the Hyades shown by a circle), shows a remarkable concentration in the (X, Y) plane with the characteristic vertex deviation. In fact, when comparing Fig. 22 with Fig. 4 of the paper of Vyssotsky (1957) and taking into account the different solar motion used in the two cases, the stars of group A appear to be located mainly within the region of the early A-type stars (although this region is not entirely occupied). Group B exhibits much less concentration in the (X, Y) plane. The (X, Y) distribution of the two groups gives a picture similar to the one found by Vyssotsky, except for the relatively small number of stars in group B.

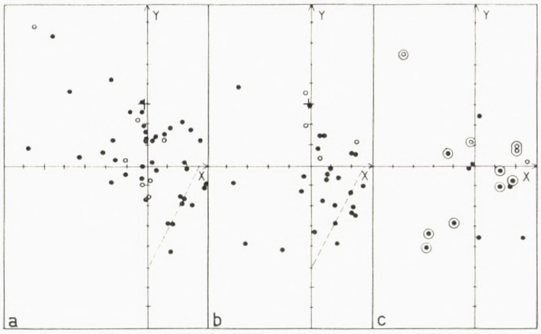

Fig. 23. Velocity components in the galactic plane for subgroups of group A: (a) stars above the Hyades level in the colour-magnitude diagram, (b) stars between the Hyades and NGC 752, (c) stars below the NGC 752 level. The open circles are stars with $|Z| > 15$ km/sec; the dots and open circles surrounded by large circles correspond to stars below or to the right of the M 67 sequence.

Following STRÖMGREN (1963a) we have attempted a subdivision of group A into age groups as follows: a) the stars in the $(B-V, M_v)$ diagram located above the Hyades line and its extension, b) the stars between this line and the corresponding NGC 752 level and c) the stars below the NGC 752 sequence and its extension. The ages of these clusters, 4×10^8 years and 10^9 years, provide approximate age limits for the three groups. Although they are somewhat different from STRÖMGREN's limits for the main-sequence stars and much less precisely defined, similar features as those found by STRÖMGREN may be detected also in the (X, Y) diagram in Fig. 23. The youngest giants (a) show a tendency of concentration close to the Ursa Major stream line (dotted in the figures) and also some alignment along the Y-axis with the head at the Hyades (cross); for the main-sequence stars the corresponding concentration appears to be slightly curved or tilted. For the older stars in Fig. 23b we again find an indication of a concentration along the Ursa Major line, as STRÖMGREN found for the 400–600 million years main sequence and a concentration along the Y-axis. The lower left quadrant is not as sparsely populated as this quadrant in the three

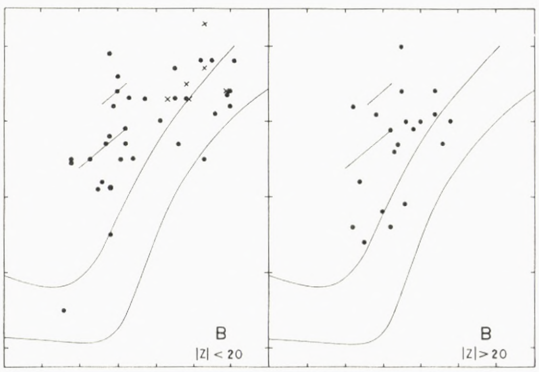

Fig. 24. Colour-magnitude diagrams for stars of group B divided into two subgroups according to the $|Z|$ value.

STRÖMGREN diagrams. These indications are only weak and additional data are needed. It should be noticed that among the three stars with space velocity larger than 63 km/sec in Fig. 23a, two are in the region of high-velocity stars originated at the apogalacticon according to VAN WIJK (1956).

Since the stars of group B are probably all comparatively old, any evolutionary and kinematic significance of the velocity distribution of these stars would be different from the one found for the young stars of group A. We divide the stars in group B into two subgroups according to $|Z|$ with the limit 20 km/sec. The colour-magnitude diagrams (Fig. 24) indicate some age division. The stars with the largest $|Z|$ values are preferably distributed around the M 67 sequence, while the stars with small

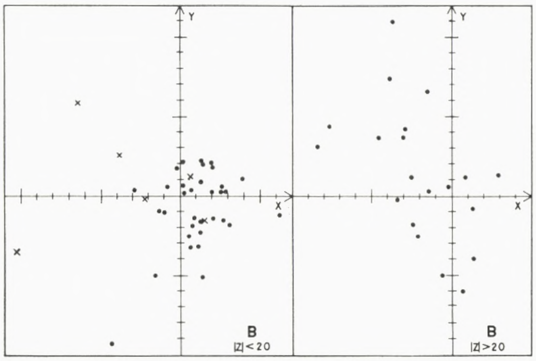

Fig. 25. (X, Y) diagram for stars of group B divided into two subgroups according to the $|Z|$ value.

$|Z|$ values are located elsewhere, mainly in three groups. The 20 stars with $|Z| > 20$ km/sec have approximately the same value of C, on the average 0.350. These stars may have some vertex shift (Fig. 25), but large as well as moderate $|Z|$ values, and high as well as low Q values are included in this grouping. The star with the highest space velocity is among those close to the Hyades in the $(B-V, M_v)$ diagram. The stars with $|Z| < 20$ represent quite different C values. The crosses in the diagrams of these stars are those with the smallest C values, on the average 0.328.

The five dot stars with $|Z| < 20$, close to NGC 188 in Fig. 24 and thus apparently very old, show similarities in their velocity data and have almost the same C, about 0.360. This indicates some kind of grouping. However, two of these stars have residuals $M_v(k, n, m) - M_v(K)$ of about $+1^m$ and one has $\Delta(B-V) = +0.06$ (cf. the duplicity estimate in Section 5); a further discussion requires more accurate data.

Among the subgiants in groups A and B, three are common with the investigation of PAGEL (1963). For these stars, γ Cep, η Cep and β Aql, no appreciable difference is noted for [Fe/H] while the ultraviolet excess is 0.00, 0.06 and 0.07 respectively. These results are in close relative agreement with the C values which are 0.399, 0.365 and 0.368 respectively.

Concluding the discussion of this section we note that the division of the giants and subgiants into two main groups with the described properties is interpreted satisfactorily in the light of current theories of stellar evolution. The separation is made by means of the third parameter without any correction, and, in fact, the division implies that a correction depending strongly on S is very improbable. If we, tentatively, repeat the above analysis using a quantity $C' = C + 0.5\ S$ instead of C (cf. p. 30), we find a (Z, C') diagram with a much less pronounced limit in the dispersion of Z, and a larger scatter in C' for the stars with $|Z|$ larger than 35 km/sec. Even more convincingly, if we attempt a division into two groups we find young giants in both groups whatever the separation value of C' within reasonable limits, and, as a consequence of the dependence of C' on S, the two groups are restricted almost exclusively to two separated intervals of $(B-V)$. This result is, of course, quite unlikely from an evolutionary point of view.

8. Summary

An empirical three-dimensional spectral classification of bright field giant and subgiant stars of the range G5–K3.5 has been established by means of the classification indices k, n and m, related to a region close to the K-line of ionized Ca, to the cyanogen absorption at 4200 A, and to the metallic line effect on the continuum, respectively. The classification parameters, S for the spectral type, L for the luminosity, and a third parameter C, have turned out to represent other spectrophotometric quantities for the stars in question with good accuracy, and absolute magnitudes $M_v(k, n, m)$ derived from L and S agree with $M_v(\mathrm{K})$ determined from the K- and H-line reversals with a scatter corresponding to a mean error $\pm 0^{\mathrm{m}}.5$ for the two sets of magnitudes. The classification parameters have further been related to photoelectric colour indices, and it was found that $(U-B)$ depends only on S, while $(B-V)$ depends on S and on C as well. The remaining scatter of these relations may be due to duplicity or to absorption by interstellar matter, and evidence of interstellar reddening is indicated for two of the program stars. The classification parameters are quite sensitive to duplicity effects. However, duplicity should be detectable, at least for spectral types earlier than K2, when measuring the G-band index in addition to the other classification indices. The detectability may be sharpened if the sensitivity of the indices can be increased by means of filters with smaller band widths; observations of known binaries consisting of giant stars would be a valuable test.

The variation of the third parameter C is assumed to be related mainly to abundance differences. The distribution of the parameter C for the program stars and its relation to the distribution of the stars in the colour-magnitude diagram and to the galactic velocity components have been studied. It is found that the value $C = 0.38$, close to the average value for the program stars, divides the data into two main groups so that a considerable fraction of the stars in the group with $C > 0.38$ are giants evolved

from A-type main-sequence stars, while the group with $C < 0.38$ consists probably chiefly of stars evolved from the F- and G-type section. The dispersion in C for A-type giants of the first group is significant, a result which should be compared with the variation of the index m_1 found by Strömgren for the A stars. This indicates that effects other than the abundance may influence the third parameter. Duplicity may contribute to this dispersion in C. On the other hand, the results of the discussion in Section 7 implies that for the giants evolved from the F- and early- G-type main-sequence, C should represent the chemical composition; a possible correction depending on S should be small.

It is important to extend this investigation to a large number of near-by stars of the types studied and, if possible, also of the late-type giants. However, as already emphasized in the introduction, our classification system is to be considered as a provisional one, giving a first approach along these lines in the age and population discussion of the giant stars. It is desirable to attempt an improvement of the classification accuracy, and we shall mention some results of the investigations in progress for the filter method.

It appeared through measurements made by Crawford (1961) and measurements made at this observatory that halving the width of the cyanogen filter increases the sensitivity of the index n considerably. The peaks of the filters of the new and the old index are also somewhat different. The two indices are correlated with a scatter which corresponds to an external mean error of the two series of $\pm 0\overset{m}{.}014$ when K4 and K5 stars are excluded. These stars and a few other stars, for instance γ Tau (peculiar, cf. p. 46), clearly exhibit systematic deviations. When comparing the present n with the Cambridge CN ratio a considerable scatter is also found, and systematic deviations for the K4 and K5 stars are even much more pronounced. In connection with the comparison of different series of cyanogen measurements we again refer to the work of Yoss (1962).

The systematic deviations for the K4 and K5 stars indicate that the relation between two cyanogen indices is also dependent on the spectral type. Generally, each of the three classification indices depends on all three parameters, and slight changes of the transmission bands corresponding to the indices will have no serious effect if the new indices, in common, define the three parameters without any new scatter. The relations between different series of measurements of an index will be discussed in connection with an account of the definite classification system.

The step from the present to an improved n-index may contribute to an increased accuracy of M_v and of the third parameter. This may, however, require also improvement of the other indices in our classification parameters. A third cyanogen index with peaks defined as for the second index but with small band width of both filters (about 40 A) is now being investigated. In this connection we measure also a new index for the G-band, defined by the long wave length intensity of the cyanogen index and an intensity for a region of similar width at 4360 A. It should be noticed here that it has not been profitable to replace k by the index g already available

(Papers I and II) in the classification system used above. A second index g measured together with the second n used only slightly differing filter widths and has appeared to be only slightly more sensitive than the first g.

A possible improvement of the sensitivity of the other indices is still questionable. A metallic-line index m_1 obtained through Strömgren's four colour-system is linearly correlated with the present index m and very nearly has the same sensitivity. Some remaining scatter is present. An index with the same peaks as the four-colour m_1 but smaller widths will be tested.

As already mentioned the significance of the index k is not quite clear. Several series of standard observations of k have been carried out utilizing the original 3920 filter together with the 4070 filter or other filters of similar half width but with peak wave lengths differing by 10–30 A. Also different phototubes have been used. These series are in agreement within the adopted measurement accuracy ($\pm 0^{m}.010$) and show only weakly varying sensitivity of the index. Thus the original index k could be used in an extended program, but attempts to reproduce this index by means of other filters may meet difficulties. Furthermore, in case of fainter stars the effect of interstellar reddening would not be well defined, and we test indices with filters of smaller widths centred on different wave lengths in the surrounding of 3920 A (cf. p. 28).

In this connection we may notice that it has been difficult to obtain filters at these wave lengths with small width and fair peak transmission. In the cyanogen- and G-band regions filters can now be produced with half widths of about 40 A, very precise peak wave lengths and peak transmissions of 50–70 per cent. Reduction of the width is then at least partly compensated by a doubling of the transmission, and the limiting magnitude would not be much changed. For the present equipment it has been $7–7^1/_2$ mag. (one minute exposure time; no cooling),

When attempting an improvement and an extension of the classification one should stress the importance of adding observations of other equivalents such as the Cambridge intensity ratios. For instance we recall the luminosity sensitivity of the FeI lines at 5250 A and the relation of the D-line intensity to our third parameter. Observations of different features covering limited regions of the spectra, strong lines and bands, combined with observations of the four-colour system might throw light on different questions raised through the preceding analysis, such as the significance of the dispersion of C for the young giants.

A powerful tool in selecting the most effective system of equivalents may be a scanning of the spectra such as the one carried out by Meinel and Golson (1959).

Finally, we may emphasize the importance of group stars and cluster stars as objects for calibrating absolute magnitudes and possibly also for establishing a precise relation between the third parameter and the spectral-type parameter on one hand, and the chemical composition and the temperature on the other hand. Abundance determinations needed for a calibration of the composition equivalent are in progress

(Wallerstein, private communication). Abundances of extreme population II giants relative to abundances of normal giants are available through the work of Wallerstein, Greenstein, Parker, Helfer and Aller (1963).

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Copenhagen University Observatory. Brorfelde.

Table 11.

The metallic-line index *m* together with the number of observation nights, listed according to the Harvard-Revised-Photometry or Bright-Star number.

The MK spectral types were given in the catalogue of Paper II (Gyldenkerne 1958a).

In the Remarks the sources of the *U*, *B*, *V* observations are indicated (cf. also p. 19):

A	Argue (1963)	MHJ	Morgan, Harris and Johnson (1953)
E	Eggen (1955)	R	Roman (1955)
J	Johnson (1955)	SK	Stebbins and Kron (1956)
JK	Johnson and Knuckles (1957)	W	Wilson and Bappu (1957)
JM	Johnson and Morgan (1953)	H	D. L. Harris, private communication

In the Remarks further SB denotes spectroscopic binary and an asterisk denotes peculiarity. The latter has been discussed in Section 5 (Table 6) for some stars. The remaining peculiarities are commented as follows:

HR		
1346	γ Tau	The star is found to be variable (Johnson and Harris 1954). The index *k* may be variable (Paper I, p. 28). The star has a discrepancy between the present *n* and a new *n*-index (cf. p. 43), and a large residual in the *W*, *S*, *C* relation (p. 27).
1409	ε Tau	The index *k* may be variable (cf. Paper I, p. 28).
3905	μ Leo	Large *b*, *S*, *L* residual.
4695	16 Vir	Peculiar location in the *u*, *C* diagram, Fig. 18.
8498	1 Lac	Large *b*, *S*, *L* residual.

HR		*m*		Remarks
163......	ε And	.264	3	R
165......	δ And	.648	3	W
166......	54 Psc	.424	2	J
168	α Cas	.449	12	SK
175......	32 And	.332	3	JK
253......	$υ^1$ Cas	.542	3	
265......	$υ^2$ Cas	.342	3	
285......		.550	2	SK
294......	ε Psc	.341	3	E
321......	μ Cas	.239	3	J
351......	χ Psc	.370	3	
352......	τ Psc	.469	3	
360......	φ Psc	.423	2	SB
390......	ξ And	.468	3	
399......	ψ Cas	.438	3	
430......	49 And	.398	2	
434......	μ Psc	.654	3	R
437......	η Psc	.348	3	J
442......	χ Cas	.347	3	
458......	υ And	.204	2	JM
464......	51 And	.603	12	J
469......	χ And	.308	3	
483......		.236	3	J
489......	ν Psc	.649	2	
493......	107 Psc	.396	3	J

(continued)

TABLE 11 (continued).

HR		m		Remarks
510	o Psc	.352	3	
511		.371	3	MHJ
549	ξ Psc	.355	3	
592	49 Cas	.354	2	
617	α Ari	.491	14	J
643	60 And	.707	2	
645		.320	3	R SB
694	64 And	.378	3	
699	65 And	.757	3	
743		.337	2	
800	14 Per	.268	3	
824	39 Ari	.472	2	
843	17 Per	.796	2	
882	24 Per	.543	2	
918		.386	2	
937	ι Per	.224	2	J
941	ϰ Per	.403	2	
947	ω Per	.452	2	
951	δ Ari	.433	3	
969		.305	3	SB
999		.669	2	
1017	α Per	.140	9	JM
1030	o Tau	.300	3	J
1052	σ Per	.649	3	
1135	ν Per	.131	3	
1256	37 Tau	.434	3	
1303	μ Per	.240	2	
1327		.275	2	JK
1343	54 Per	.377	3	
1346	γ Tau	.390	3	J *
1348	φ Tau	.483	3	
1373	δ Tau	.402	2	J
1396	π Tau	.350	3	
1407	75 Tau	.519	3	
1409	ε Tau	.407	3	J *
1411	θ^1 Tau	.366	3	
1457	α Tau	.796	3	J
1523		.611	3	
1577	ι Aur	.645	6	W
1580	o^2 Ori	.485	2	
1603	β Cam	.262	3	
1729	λ Aur	.237	2	JM

HR		m		Remarks
1739	109 Tau	.360	2	A
1907	φ^2 Ori	.332	3	R
1963	51 Ori	.461	2	R
1995	τ Aur	.352	2	
2002	132 Tau	.416	2	A *
2012	ν Aur	.475	2	
2047	χ^1 Ori	.224	2	JK
2077	δ Aur	.399	6	J
2152	37 Cam	.391	3	
2219	ϰ Aur	.374	2	R
2289	ψ^1 Aur	.450	1	
2427	ψ^2 Aur	.540	2	R
2459	ψ^4 Aur	.791	2	R
2473	ε Gem	.465	2	A
2477	13 Lyn	.378	2	
2478	30 Gem	.482	3	
2487	ψ^6 Aur	.442	2	A
2506	18 Mon	.455	2	
2516	ψ^7 Aur	.562	2	R
2527		.744	2	
2649		.651	3	A
2697	τ Gem	.563	2	A
2715	18 Lyn	.471	2	
2793	65 Aur	.414	2	
2805	66 Aur	.471	2	A
2808	57 Gem	.363	2	A
2821	ι Gem	.410	3	H
2864	6 CMi	.541	3	
2990	β Gem	.435	3	J
3149	χ Gem	.495	2	A SB
3176	μ Cnc	.261	2	H
3249	β Cnc	.712	2	J
3275	31 Lyn	.790	2	H
3403	π^2 UMa	.519	2	SK
3418	σ Hya	.523	2	
3461	δ Cnc	.453	2	H
3508	35 Lyn	.345	11	A
3522	ϱ^1 Cnc	.459	1	A
3547	ζ Hya	.381	2	H
3612		.356	3	A
3627	ξ Cnc	.371	2	A SB
3731	ϰ Leo	.578	2	A
3751		.675	3	

(continued)

Table 11 (continued).

HR		m		Remarks
3771......	24 UMa	.265	2	
3773......	λ Leo	.798	2	A
3800......	10 LMi	.341	3	H
3815......	11 LMi	.369	2	J
3839......	27 UMa	.405	2	
3851......	43 Lyn	.355	2	A
3873......	ε Leo	.264	2	SK–A
3905......	μ Leo	.584	2	H *
4126......		.379	3	
4178......	38 UMa	.560	2	SB
4181......		.679	2	
4246......	44 UMa	.589	2	A
4247......	46 LMi	.439	11	H
4258......	46 UMa	.460	2	H
4291......	58 Leo	.485	2	H
4301......	α UMa	.402	8	J *
4335......	ψ UMa	.490	7	J
4365......	73 Leo	.503	2	A SB
4377......	ν UMa	.610	4	J
4461......	2 Dra	.355	2	
4495......	92 Leo	.394	2	A
4496......	61 UMa	.324	3	JM
4504......	3 Dra	.556	2	
4518......	χ UMa	.474	6	H
4521......		.632	3	A
4608......	o Vir	.348	3	H *
4667......	7 Com	.361	3	H
4668......		.470	3	H SB
4695......	16 Vir	.454	2	R *
4697......	11 Com	.374	3	H
4716......	5 CVn	.319	3	H
4728......	6 CVn	.391	2	H
4737......	γ Com	.516	6	H
4783......		.392	2	A
4785......	β CVn	.199	2	J
4883......	31 Com	.212	3	A
4928......	9 Dra	.491	2	R
4932......	ε Vir	.370	2	H
4954......	41 Com	.773	2	H
4983......	β Com	.223	4	J
4997......		.419	3	H
5013......		.648	4	A
5072.....	70 Vir	.278	3	J
5200.....	υ Boo	.804	4	H
5201.....	6 Boo	.672	2	H
5247.....	9 Boo	.688	2	H
5330.....	15 Boo	.378	2	A
5340.....	α Boo	.514	13	J
5370.....	20 Boo	.609	2	A
5429.....	ϱ Boo	.613	4	J
5430.....	5 UMi	.700	3	
5480.....	31 Boo	.387	2	A *
5502.....	o Boo	.371	2	A
5541.....		.410	2	R
5563.....	β UMi	.721	4	MHJ
5602.....	β Boo	.349	12	MHJ
5681.....	δ Boo	.343	2	J
5744.....	ι Dra	.548	2	J
5854.....	α Ser	.547	2	J
5901.....	$\varkappa$ CrB	.453	2	H
5947.....	ε CrB	.539	2	J
5966.....	5 Her	.383	2	R
6018.....	τ CrB	.440	3	MHJ SB
6103.....	ξ CrB	.416	2	A
6126.....		.483	2	
6132.....	η Dra	.337	2	MHJ
6199.....		.442	2	A
6220.....	η Her	.332	3	A
6299.....	$\varkappa$ Oph	.529	2	MHJ
6603.....	β Oph	.552	2	J
6688.....	ξ Dra	.551	6	MHJ
6703.....	ξ Her	.368	3	H
6705.....	γ Dra	.776	6	J
6872.....	$\varkappa$ Lyr	.497	2	A
6895.....	109 Her	.524	2	R
6945.....	42 Dra	.466	2	
7137.....		.296	2	
7180.....	υ Dra	.461	2	
7295.....	53 Dra	.395	3	A
7309.....	54 Dra	.530	2	A
7310.....	δ Dra	.372	5	MHJ
7328.....	$\varkappa$ Cyg	.364	2	MHJ
7352.....	τ Dra	.616	3	SB
7462.....	σ Dra	.349	4	J
7468....		.387	3	A

(continued)

Table 11 (continued).

HR		m		Remarks
7517......	15 Cyg	.336	3	A
7576......	20 Cyg	.653	3	E
7602......	β Aql	.323	3	JM
7615......	η Cyg	.415	8	H
7685......	ϱ Dra	.599	2	
7796......	γ Cyg	.185	1	
7806......	39 Cyg	.641	3	R
7949......	ε Cyg	.400	9	J SB
7955......		.199	1	
7957......	η Cep	.353	3	J
8085......	61 Cyg A	.778	3	J
8173......	1 Peg	.477	2	J
8228......	71 Cyg	..393	2	
8232......	β Aqr	.248	3	JM
8252......	ϱ Cyg	.313	2	JK
8255......	72 Cyg	.449	3	R
8308......	ε Peg	.546	3	JM
8313......	9 Peg	.352	3	JM
8317......	11 Cep	.500	3	
8324......		.469	2	
8414......	α Aqr	.307	3	JM
8465......	ζ Cep	.531	13	SB
8468.....	24 Cep	.300	3	
8475.....		.488	2	
8485.....		.550	5	SB
8498.....	1 Lac	.606	3	*
8538.....	β Lac	.381	2	
8551.....	35 Peg	.422	3	R
8632.....	11 Lac	.573	2	
8656.....	13 Lac	.373	2	
8684.....	μ Peg	.351	3	SK
8694.....	ι Cep	.426	3	
8702.....		.622	2	SK
8779.....		.573	3	
8780.....	3 And	.412	2	
8796.....	56 Peg	.514	2	SB
8832.....		.615	3	J
8852.....	γ Psc	.291	3	R
8874.....	11 And	.395	3	
8916.....	θ Psc	.447	3	E
8923.....	70 Peg	.362	3	
8930.....	14 And	.402	3	R
8974.....	γ Cep	.487	9	J
9008.....	τ Cas	.465	3	

Table 12.

HR	S	L	C	M_v	π	X	Y	Z	Q
163....	.359	.186	.346	$+1^{m}.5$	$0''.027$	− 44	− 93	+ 19	105
165....	1.055	.179	.423	+ 0.1	.023	− 13	+ 6	− 5	15
166....	.542	.073	.379						
168....	.898	.252	.400	− 1.9	.014	+ 2	+ 2	− 2	3
175....	.520	.174	.379	+ 1.5	.017	+ 13	− 16	+ 9	22
253....	.919	.174	.389	+ 0.6	.014	+ 1	− 32	− 4	32
265....	.555	.162	.341	+ 1.8	.027	− 16	− 50	+ 3	53
285....	.996	.218	.406	− 1.0	.0085	− 3	+ 32	+ 9	33
294....	.579	.177	.344	+ 1.2	.024	+ 31	− 18	+ 4	36
351....	.701	.227	.400	− 0.7	.0082	+ 8	+ 16	− 4	18
352....	.800	.170	.379	+ 1.0	.020	+ 21	+ 18	− 14	31
360....	.716	.179	.411						
390....	.833	.191	.383	+ 0.2	.012	+ 1	− 7	+ 17	18

(continued)

Table 12 (continued).

HR	S	L	C	M_v	π	X	Y	Z	Q
399....	.768	.190	.397	$+0^{\mathrm{m}}.4$	$''.013$	− 11	+ 3	+ 21	24
430....	.717	.203	.395	+ 0.1	.0097	− 1	− 16	− 8	18
437....	.622	.212	.397	0.0	.019	+ 18	+ 2	− 3	18
442....	.628	.200	.358	+ 0.4	.014	+ 27	− 15	− 1	31
464....	1.017	.188	.414	− 0.1	.018	+ 8	+ 13	− 20	25
469....	.476	.192	.391	+ 1.0	.016	+ 25	− 10	+ 9	28
489....	1.056	.162	.364	+ 0.6	.016	+ 21	− 14	+ 7	26
493....	.452	.049	.359						
510....	.630	.209	.397	+ 0.1	.014	+ 17	+ 22	+ 12	30
511....	.370	.061	.400						
549....	.612	.193	.387	+ 0.6	.015	+ 26	+ 13	− 12	31
592....	.591	.191	.404	+ 0.8	.014	+ 19	− 13	+ 1	23
617....	.853	.174	.359	+ 0.7	.053	− 10	− 10	+ 11	18
645....	.599	.241	.410						
694....	.655	.189	.389	+ 0.7	.012	− 5	− 12	0	13
743....	.530	.181	.401	+ 1.2	.016	+ 20	− 20	+ 10	30
800....	.400	.237	.418						
824....	.861	.199	.372	− 0.1	.012	− 58	+ 9	+ 5	59
882....	.945	.184	.374	+ 0.2	.012	+ 14	− 51	+ 13	54
918....	.714	.205	.374	0.0	.010	+ 7	− 60	+ 31	68
941....	.714	.196	.396	+ 0.3	.020	− 18	+ 43	− 8	47
947....	.829	.206	.387	− 0.3	.010	+ 28	− 11	+ 3	30
951....	.714	.161	.391	+ 1.4	.026	+ 2	+ 25	+ 8	26
969....	.613	.264	.401						
1030....	.535	.219	.368	0.0	.019	+ 11	− 42	0	43
1052....	1.053	.157	.357	+ 0.8	.019	+ 26	+ 3	+ 11	28
1256....	.792	.203	.387	− 0.1	.013	− 22	+ 7	+ 12	26
1327....	.409	.218	.403	+ 0.3	.010	+ 11	− 28	− 6	31
1343....	.617	.183	.421	+ 1.0	.016	+ 12	− 38	+ 7	40
1346....	.637	.174	.412	+ 0.68	.0253	− 2	+ 31	+ 4	31
1373....	.718	.203	.408	+ 0.66	.0239	− 2	+ 31	+ 5	31
1396....	.650	.220	.389	− 0.3	.0096	+ 4	+ 15	− 15	22
1407....	.845	.152	.393	+ 1.4	.018	+ 20	+ 9	+ 6	23
1409....	.734	.209	.414	+ 0.54	.0253	− 1	+ 31	+ 5	31
1411....	.617	.200	.428	+ 0.80	.0245	− 2	+ 32	+ 4	32
1523....	1.000	.158	.369	+ 0.9	.014	+ 13	− 23	+ 10	28
1580....	.863	.174	.323	+ 0.7	.021	+ 17	− 15	− 12	26
1603....	.363	.234	.426						
1739....	.558	.170	.413	+ 1.5	.020	− 4	+ 7	− 3	9
1907....	.481	.141	.346	+ 2.6	.050	− 39	+ 74	− 20	86
1963....	.791	.159	.340	+ 1.3	.019	− 15	+ 66	− 30	74
1995....	.551	.170	.398	+ 1.5	.025	+ 11	− 28	− 1	30

(continued)

TABLE 12 (continued).

HR	S	L	C	M_v	π	X	Y	Z	Q
2002....	.625	.169	.492						
2012....	.848	.196	.392	$0^m.0$	$''.016$	+ 19	− 1	+ 8	21
2077....	.702	.183	.367	+ 0.8	.026	− 8	+ 6	+ 12	16
2152....	.697	.187	.365	+ 0.6	.012	+ 29	+ 13	+ 25	40
2219....	.650	.170	.334	+ 1.3	.025	− 25	+ 12	− 27	39
2289....	1.021	.333	.432						
2427....	.977	.208	.381	− 0.7	.0079	− 17	+ 13	+ 3	22
2473....	1.056	.336	.425						
2477....	.608	.156	.367	+ 1.9	.021	+ 13	+ 9	+ 16	22
2478....	.885	.202	.366	− 0.3	.011	− 11	− 4	− 5	13
2487....	.824	.218	.399	− 0.6	.0072	+ 17	− 18	+ 5	25
2506....	.798	.183	.382	+ 0.5	.016	+ 9	− 1	0	9
2516....	.998	.205	.395	− 0.6	.0075	− 55	+ 69	− 19	90
2697....	.979	.195	.397	− 0.3	.012	− 1	+ 14	− 3	14
2715....	.786	.163	.391	+ 1.2	.016	− 35	+ 55	− 20	68
2793....	.691	.162	.373	+ 1.5	.020	+ 15	+ 20	− 5	25
2805....	.858	.199	.374	− 0.1	.0090	+ 2	+ 13	+ 8	15
2808....	.455	.113	.407	+ 3.6	.054	+ 13	− 2	+ 3	13
2821....	.693	.168	.373	+ 1.3	.033	+ 7	+ 4	− 10	13
2864....	.987	.213	.380	− 0.9	.0076	+ 10	− 28	− 2	30
2990....	.696	.152	.399	+ 1.0	.093	+ 20	+ 7	− 20	29
3149....	.840	.178	.412						
3403....	.860	.149	.356	+ 1.5	.024	+ 26	+ 6	+ 6	27
3418....	.937	.209	.418	− 0.6	.010	− 6	+ 8	+ 6	12
3461....	.787	.179	.388	+ 0.7	.022	− 38	− 8	− 2	39
3508....	.581	.176	.354	+ 1.3	.018	+ 29	+ 3	+ 14	32
3522....	.549	.056	.430						
3547....	.654	.189	.400	+ 0.7	.032	+ 4	+ 15	+ 8	17
3612....	.669	.227	.400						
3627....	.674	.215	.414						
3731....	.931	.149	.390	+ 1.3	.023	− 2	+ 12	+ 20	23
3800....	.522	.176	.408	+ 1.4	.024	+ 11	− 19	− 1	22
3815....	.383	.068	.401						
3839....	.767	.227	.406	− 0.8	.0062	− 16	+ 3	− 7	18
3851....	.592	.191	.408	+ 0.8	.012	− 5	+ 23	+ 17	29
3873....	.351	.219	.409						
3905....	1.023	.212	.425						
4126....	.648	.187	.394	+ 0.8	.015	+ 21	+ 7	+ 14	26
4178....	.969	.196	.413						
4181....	1.038	.134	.383	+ 1.6	.020	+ 12	− 10	+ 9	18
4246....	1.032	.197	.375	− 0.4	.0076	− 3	+ 27	− 8	28
4247....	.748	.160	.350	+ 1.4	.034	− 21	− 25	+ 29	44

(continued)

Table 12 (continued).

HR	S	L	C	M_v	π	X	Y	Z	Q
4258....	.789	.168	.368	$+1^m.1$	$''.016$	− 2	+ 6	− 26	27
4291....	.815	.160	.371	+ 1.3	.019	+ 9	− 14	+ 12	21
4301....	.658	.185	.449	+ 0.8	.063	+ 4	− 7	− 1	8
4335....	.855	.183	.385	+ 0.4	.030	+ 7	− 4	+ 1	8
4365....	.664	.095	.496						
4377....	1.010	.172	.395	+ 0.4	.023	+ 18	− 7	− 4	19
4461....	.613	.186	.367	+ 0.9	.014	− 6	− 50	+ 45	68
4495....	.618	.151	.392	+ 2.0	.022	− 2	− 1	+ 11	11
4504....	.970	.191	.381	− 0.1	.0082	+ 21	+ 18	− 9	29
4518....	.854	.182	.339	+ 0.5	.023	+ 6	+ 13	− 9	17
4521....	1.038	.185	.440	− 0.1	.0085	+ 3	− 21	+ 19	28
4608....	.477	.140	.398	+ 2.7	.053	+ 20	+ 9	− 23	32
4667....	.574	.160	.371	+ 1.8	.024	+ 13	− 8	− 21	26
4668....	.818	.170	.349						
4695....	.823	.178	.320	+ 0.7	.014	− 65	+ 59	+ 18	90
4697....	.589	.148	.361	+ 2.2	.031	+ 8	+12	+ 49	51
4716....	.542	.205	.389	+ 0.4	.013	+ 16	− 15	− 6	23
4728....	.604	.157	.416	+ 1.8	.022	− 1	+ 1	+ 4	4
4737....	.913	.201	.412	− 0.3	.012	− 33	+ 5	+ 8	34
4783....	.666	.176	.378	+ 1.1	.015	+ 6	− 25	− 10	28
4928....	.872	.176	.336	+ 0.6	.011	− 39	+ 26	− 10	48
4932....	.586	.175	.422	+ 1.3	.050	+ 5	+ 15	− 5	17
4997....	.762	.213	.418	− 0.3	.0092	− 1	+ 13	− 12	18
5330....	.633	.180	.393	+ 1.0	.014	− 32	− 38	+ 10	51
5340....	.905	.167	.315	− 0.3	.090	−103	− 35	+ 4	109
5370....	.988	.167	.420	+ 0.6	.014	− 2	+ 36	+ 19	41
5429....	1.020	.165	.355	+ 0.6	.025	+ 14	+ 22	+ 2	26
5480....	.621	.186	.454	+ 0.9	.016	+ 13	+ 3	− 11	17
5502....	.615	.176	.389	+ 1.2	.022	− 4	− 5	+ 2	7
5541....	.639	.136	.364	+ 2.4	.024	− 29	+ 42	− 35	62
5602....	.604	.195	.385	+ 0.6	.027	− 3	− 6	− 5	8
5681....	.549	.159	.347	+ 1.9	.051	+ 8	− 19	− 7	22
5744....	.920	.172	.398	+ 0.6	.029	+ 7	− 7	0	10
5854....	.926	.189	.438	+ 0.1	.032	+ 34	− 17	0	38
5901....	.696	.132	.397	+ 2.4	.036	− 24	− 40	− 11	48
5947....	.951	.179	.336	+ 0.3	.018	− 23	− 1	− 5	24
5966....	.583	.138	.370	+ 2.5	.030	+ 20	+ 21	+ 5	29
6018....	.662	.128	.401						
6103....	.684	.171	.409	+ 1.2	.021	− 1	+ 30	+ 5	30
6126....	.869	.208	.417	− 0.4	.0073	− 3	− 9	+ 17	19
6132....	.572	.196	.390	+ 0.6	.038	+ 4	− 2	− 2	5
6199....	.679	.147	.434	+ 2.0	.022	+ 2	+ 5	− 6	8
6220....	.521	.175	.380	+ 1.4	.039	+ 19	− 23	+ 9	31

(continued)

TABLE 12 (continued).

HR	S	L	C	M_v	π	X	Y	Z	Q
6299....	.890	.170	.400	$+0^{m}.8$	″.032	− 35	+ 39	+ 13	54
6603....	.925	.180	.427	+ 0.4	.033	+ 22	+ 12	+ 19	31
6688....	.935	.168	.360	+ 0.7	.025	+ 2	+ 7	− 19	20
6703....	.546	.154	.416	+ 2.1	.049	+ 17	− 10	− 1	20
6872....	.920	.216	.389	− 0.8	.010	− 1	+ 17	+ 13	21
6895....	.888	.164	.366	+ 1.0	.027	− 34	− 2	− 54	64
6945....	.837	.188	.359	+ 0.3	.012	+ 62	− 12	− 12	64
7137.. .	.467	.204	.388	+ 0.6	.014	+ 22	− 19	+ 8	30
7180....	.862	.215	.384	− 0.6	.0085	+ 11	+ 19	− 14	26
7295....	.661	.185	.426	+ 0.8	.014	+ 3	+ 10	− 3	11
7309....	.861	.155	.406	+ 1.3	.017	− 10	− 28	− 4	30
7310....	.666	.190	.359	+ 0.6	.032	+ 39	+ 11	+ 9	42
7328....	.653	.204	.388	+ 0.2	.019	− 9	+ 27	− 3	29
7352....	1.026	.192	.450						
7462....	.360	.077	.379						
7468....	.609	.154	.392	+ 1.9	.024	+ 2	− 35	+ 15	38
7517....	.610	.210	.373	+ 0.1	.011	+ 2	+ 22	− 15	27
7576....	1.057	.189	.468	− 0.2	.0091	− 4	− 43	− 7	44
7602....	.392	.121	.368	+ 3.5	.091	− 29	+ 4	+ 1	29
7615....	.727	.182	.372	+ 0.7	.023	− 13	− 9	+ 10	19
7685....	1.061	.215	.400	− 1.1	.0076	− 2	+ 20	+ 12	23
7806....	1.067	.176	.366	+ 0.2	.014	+ 3	+ 2	− 5	6
7949....	.706	.175	.344						
7957....	.532	.145	.365	+ 2.4	.062	− 84	+ 31	+ 22	92
8173....	.812	.167	.376	+ 1.0	.024	− 46	+ 37	+ 28	65
8228....	.656	.172	.387	+ 1.2	.016	− 3	+ 20	+ 23	31
8232....	.397	.234	.418						
8252....	.465	.184	.401	+ 1.3	.029	+ 21	− 24	− 1	32
8255....	.805	.201	.406	0.0	.010	− 46	+ 64	+ 14	80
8313....	.780	.290	.404						
8317....	.871	.192	.415	+ 0.1	.012	− 38	+ 37	− 2	53
8324....	.790	.175	.416	+ 0.8	.013	− 14	− 41	− 1	43
8414....	.535	.223	.401						
8465....	1.064	.290	.453						
8468....	.531	.231	.395	− 0.4	.0090	− 3	0	− 1	3
8475....	.870	.195	.391	0.0	.0089	0	− 15	− 17	23
8485....	.888	.174	.474						
8498....	.932	.113	.341						
8538....	.639	.165	.357	+ 1.5	.026	+ 7	− 32	− 18	37
8551....	.679	.143	.360	+ 2.1	.029	+ 14	− 39	− 62	75
8632....	.985	.180	.365	+ 0.2	.014	− 2	+ 18	− 1	18
8656....	.678	.213	.405	− 0.1	.0092	+ 29	− 10	+ 10	32

(continued)

Table 12 (continued).

HR	S	L	C	M_v	π	X	Y	Z	Q
8684....	.575	.170	.364	$+1^{m}.5$	$.040''$	+20	+3	−11	23
8694....	.747	.182	.376	+0.7	.027	+12	−32	−7	35
8702....	1.100	.220	.401	−1.3	.0060	−18	−8	−9	22
8779....	1.006	.213	.435	−0.9	.0056	0	+6	+13	14
8780....	.718	.175	.360	+1.0	.018	−30	+37	+37	60
8832....	.682	−.022	.417						
8852....	.480	.199	.348	+0.8	.026	−37	+111	−28	120
8874....	.635	.157	.383	+1.8	.020	+25	+2	+16	30
8916....	.739	.169	.412	+1.1	.024	+23	−35	+6	42
8923....	.585	.174	.397	+1.3	.023	+4	+4	+18	19
8930....	.694	.172	.356	+1.1	.015	−77	+44	−20	91
8974....	.754	.131	.399	+2.3	.066	−23	−33	+8	41
9008....	.824	.181	.360	+0.6	.014	−14	+3	+23	27

Notes to Table 12 and Figures 20–25.

Stars within the considered limits of S and L are included (HR 8702 is outside the limits), but for SB stars (cf. Table 11) and for stars with large (b, S, L) residuals, M_v, π, X, Y, Z and Q are not published. For the SB stars HR 360, 3149, 3627, 4668 these data were computed, and the stars are included in the plots of Figures 20–25, but are excluded from the computations pertaining to Tables 8 and 10.

For HR 4301, HR 4608, HR 4695 and HR 5480 (asterisk, Table 11), and for HR 2808 ($M_v(k, n, m) - M_v(\mathrm{K}) = +2.0$) the data may be unreliable.

X, Y, Z and Q were first computed utilizing the "basic solar motion" (Vyssotsky and Janssen 1951); later +5 and +1 were added to X and Z respectively (and new Q values computed) in order to correct to the "standard solar motion". Deviations of 1 km/sec will occur in some cases between the given X, Z and Q and corresponding values computed by means of the solar motion adopted by Allen (1963). All data in Table 12 are preliminary, and improved values will probably be obtained through more accurate classification measurements and through a better calibration.

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